L6 Mathematics Mock

Paper 1 (Teacher X)
January 2022
2021-2022

Duration: 1 hour 15 minutes

Total number of marks: 60

Write your answers on file paper provided.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is given below:

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \ldots + {}^nC_r\, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1.

(i) The equation $x^2+3x+k=0$ has repeated roots. Find the value of the constant k. **[2]**

(ii) Solve the inequality $6+x-x^2>0$. **[2]**

2.

Solve the equation $3x^{\frac{2}{3}}+x^{\frac{1}{3}}-2=0$. **[5]**

3.

(i) Express $4x^2-12x+11$ in the form $a(x+b)^2+c$. **[3]**

(ii) State the number of real roots of the equation $4x^2-12x+11=0$. **[1]**

(iii) Explain fully how the value of r is related to the number of real roots of the equation $p(x+q)^2+r=0$ where p, q and r are real constants and $p>0$. **[2]**

4.

Find the equation of the normal to the curve $y=\frac{6}{x^2}-5$ at the point on the curve where $x=2$. Give your answer in the form $ax+by+c=0$, where a, b and c are integers. **[7]**

5.

A student's attempt to solve the equation $2\log_2 x-\log_2\sqrt{x}=3$ is shown below.

$2\log_2 x-\log_2\sqrt{x}=3$	
$2\log_2\left(\frac{x}{\sqrt{x}}\right)=3$	using the subtraction law for logs
$2\log_2\left(\sqrt{x}\right)=3$	simplifying
$\log_2 x=3$	using the power law for logs
$x=3^2=9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each. **(2)**

(b) Write out the correct solution. **(3)**

6.

Given

$$2^x\times 4^y=\frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

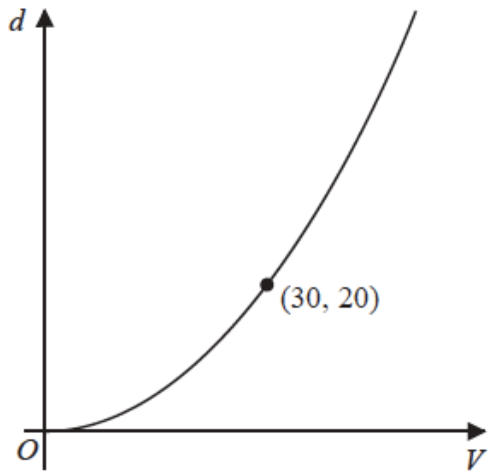
7.

A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of V km h^{-1}.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.



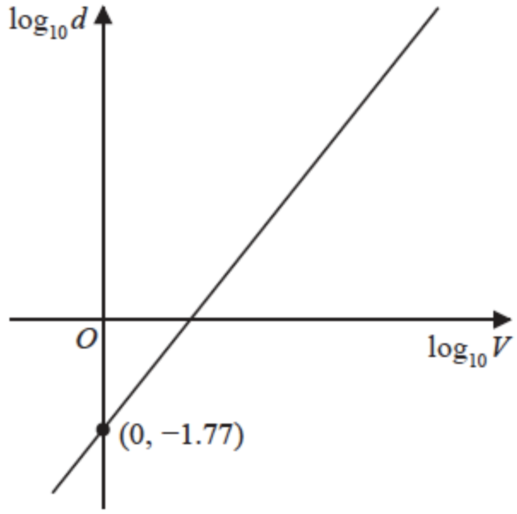


Figure 5 **Figure 6**

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

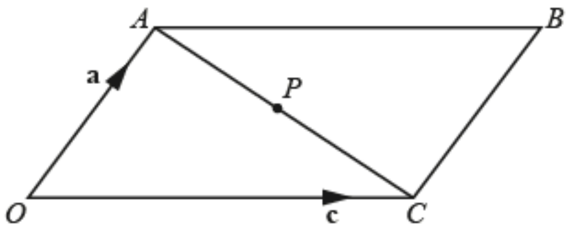
(3)

8.

Find the region of the curve $y = x^3 - 4x^2 + 6$ where the curve is decreasing. Express your answer in set notation.

9.

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is the midpoint of AC.



(i) Find the following in terms of **a** and **c**, simplifying your answers.

(a) $\overrightarrow{AC}$ **[1]**

(b) $\overrightarrow{OP}$ **[2]**

(ii) Hence prove that the diagonals of a parallelogram bisect one another. **[4]**

10.

Trees in a forest may be affected by one of two types of fungal disease, but not by both.

The number of trees affected by disease A, n_{A}, can be modelled by the formula

$$n_{\mathrm{A}} = a\mathrm{e}^{0.1t}$$

where t is the time in years after 1 January 2017.

The number of trees affected by disease B, n_{B}, can be modelled by the formula

$$n_{\mathrm{B}} = b\mathrm{e}^{0.2t}$$

On 1 January 2017 a **total** of 290 trees were affected by a fungal disease.

On 1 January 2018 a **total** of 331 trees were affected by a fungal disease.

(a) Show that $b = 90$, to the nearest integer, and find the value of a.

[3 marks]

(b) Estimate the total number of trees that will be affected by a fungal disease on 1 January 2020.

[1 mark]

(c) Find the year in which the number of trees affected by disease B will first exceed the number affected by disease A.

[3 marks]

(d) Comment on the long-term accuracy of the model.

[1 mark]