# L6 Mock Teacher X 21-22 SOLUTIONS [60]

1

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|---|---|---|---|---|---|---|
| (i) | | $3^2 - 4k = 0$<br><br>$k = \frac{9}{4}$ or 2.25 | M1<br><br>A1<br>[2] | 1.2<br><br>1.1 | $x^2 + 3x + k = (x+a)^2 = x^2 + 2ax + a^2$<br>$\Rightarrow a = 1.5 \Rightarrow k = 1.5^2$ | or $(x + 1.5)^2 - 2.25 + k = 0$ |
| (ii) | | $(3 - x)(2 + x) > 0$ or $(x - 3)(x + 2) < 0$<br>$-2 < x < 3$ or $3 > x > -2$ ISW<br>or $x \in (-2, 3)$ | M1<br>A1<br><br>[2] | 1.1a<br>2.2a | oe Allow $(3 - x)(2 + x)$ or $(x - 3)(x + 2)$<br>Allow $x > -2, x < 3$ or $x > -2$ and $x < 3$<br><br>Correct ans: BOD M1A1 | or $-2$ and 3 seen<br><br>$x > -2$ or $x < 3$ M1A0 unless followed by ans |

2.

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| 3 | *Let* $y = x^{\frac{1}{3}}$ | *M1 | Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket |
| | $3y^2 + y - 2 = 0$ | | |
| | $(3y - 2)(y + 1) = 0$ | DM1 | Correct method to find roots |
| | $y = \frac{2}{3}, y = -1$ | A1 | Both values correct |
| | $x = \left(\frac{2}{3}\right)^3, x = (-1)^3$ | DM1 | Attempt cube of at least one value |
| | $x = \frac{8}{27}, x = -1$ | A1 ft 5 | Both answers correctly followed through |
| | | 5 | SR If M1* not awarded, B1 x = -1 from T & I |

3.

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| (i) | | $4\left[x^2 - 3x\right] + 11$ | | | No marks until attempt to complete the square |
| | | $4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 11$ $\quad a = 4$ | B1 | 1.1 | Must be of the form $4(x \pm \alpha)^2 \pm \ldots$ |
| | | $(x - 3/2)^2$ | B1 | 1.1 | |
| | | $4\left(x - \frac{3}{2}\right)^2 + 2$ $\quad c = 2$ | B1<br><br>[3] | 1.1 | |
| (ii) | | No real roots | B1<br><br>[1] | 2.2a | Zero, none, 0, … if not 'no real roots' must be consistent with their (i) |
| (iii) | | $r = 0 \Rightarrow 1$ real root or 1 repeated root | M1 | 2.4 | Attempt to relate the value of $r$ to the number of real roots (this can be implied with at least one correct statement) |
| | | $r < 0 \Rightarrow 2$ real roots<br>$r > 0 \Rightarrow$ no real roots | A1<br>[2] | 2.4 | All three statements correct |

4.

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| | | $\frac{dy}{dx} = -12x^{-3}$ | M1<br>A1 | Attempt to differentiate (i.e. $kx^{-3}$ seen)<br>Correct derivative | "+ C" is A0 |
| | | When $x = 2$, $\frac{dy}{dx} = -\frac{3}{2}$ | A1 | Correct value of $\frac{dy}{dx}$. Allow equivalent fractions. | |
| | | Gradient of normal = $\frac{2}{3}$ | B1 FT | Follow through their **evaluated** $\frac{dy}{dx}$ | Must be processed correctly |
| | | When $x = 2, y = -\frac{7}{2}$ | B1 | Correct $y$ coordinate, accept equivalent forms | |
| | | $y + \frac{7}{2} = \frac{2}{3}(x - 2)$ | M1 | Correct equation of straight line through (2, their evaluated $y$), any non-zero gradient | |
| | | $4x - 6y - 29 = 0$ | A1<br>[7] | Correct equation in required form i.e. $k(4x - 6y - 29) = 0$ for integer k. **Must have "=0".** | |

5.

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| **5 (a)** | Identifies one of the two errors<br>"You cannot use the subtraction law without dealing with the 2 first"<br>" They undo the logs incorrectly. It should be $x = 2^3 = 8$ " | | B1 | 2.3 |
| | Identifies both errors. See above. | | B1 | 2.3 |
| | | | **(2)** | |
| **(b)** | $\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$ | $\frac{3}{2}\log_2(x) = 3$ | M1 | 1.1b |
| | $x^{\frac{3}{2}} = 2^3$ or $\frac{x^2}{\sqrt{x}} = 2^3$ | $x = 2^2$ | M1 | 1.1b |
| | $x = \left(2^3\right)^{\frac{2}{3}} = 4$ | $x = 4$ | A1 | 1.1b |
| | | | **(3)** | |
| | | | | **(5 marks)** |

6.

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| **1** | $2^x \times 4^y = \frac{1}{2\sqrt{2}} \quad \left\{= \frac{\sqrt{2}}{4}\right\}$ | | |
| **Special Case** | **If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of**<br>• $2^x \times 4^y \to 2^{x+2y}$ • $2^x \times 4^y \to 4^{\frac{1}{2}x+y}$ • $\frac{1}{2^x 2\sqrt{2}} \to 2^{-x-\frac{3}{2}}$<br>• $\log 2^x + \log 4^y \to x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$<br>• $\ln 2^x + \ln 4^y \to x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$<br>• $y = \log\left(\frac{1}{2^x 2\sqrt{2}}\right)$ o.e. {base of 4 omitted} | | |
| **Way 1** | $2^x \times 2^{2y} = 2^{-\frac{3}{2}}$ | B1 | 1.1b |
| | $2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$ | M1 | 2.1 |
| | E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ | A1 | 1.1b |
| | | **(3)** | |

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| **Way 2** | $\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$ | B1 | 1.1b |
| | $\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right)$ <br> $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = \ldots$ | M1 | 2.1 |
| | $y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$ | A1 | 1.1b |
| | | **(3)** | |
| **Way 3** | $\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$ | B1 | 1.1b |
| | $\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow y = \ldots$ | M1 | 2.1 |
| | $y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$ | A1 | 1.1b |
| | | **(3)** | |
| **Way 4** | $\log_2(2^x \times 4^y) = \log_2\left(\frac{1}{2\sqrt{2}}\right)$ | B1 | 1.1b |
| | $\log_2 2^x + \log_2 4^y = \log_2\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = \ldots$ | M1 | 2.1 |
| | E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(3 marks)** |

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| **Way 5** | $4^{\frac{1}{2}x} \times 4^y = 4^{-\frac{3}{4}}$ | B1 | 1.1b |
| | $4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4} \Rightarrow y = \ldots$ | M1 | 2.1 |
| | E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ | A1 | 1.1b |
| | | **(3)** | |

7.

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| **9 (a) Way 1** | $\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10} k + n\log_{10} V$ <br> or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ <br> seen or used as part of their argument | M1 | 2.1 |
| | Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$ | A1 | 2.4 |
| | $\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ <br> linked together in the same part of the question | B1 * | 1.1b |
| | | **(3)** | |
| **9 (a) Way 2** | $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ <br> or $\log_{10} d = \log_{10} k + n\log_{10} V$ <br> seen or used as part of their argument | M1 | 2.1 |
| | $\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10}(kV^n)$ <br> $\Rightarrow$ $\log_{10} d = \log_{10} k + \log_{10} V^n$ $\Rightarrow$ $\log_{10} d = \log_{10} k + n\log_{10} V$ | A1 | 2.4 |
| | $\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ <br> linked together in the same part of the question | B1 * | 1.1b |
| | | **(3)** | |
| **(a) Way 3** | Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ | M1 | 2.1 |
| | $\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ <br> **or** $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ <br> $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$ | A1 | 2.4 |
| | $\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ <br> linked together in the same part of the question | B1 * | 1.1b |
| | | **(3)** | |
| **(b)** | $\{d = 20, V = 30 \Rightarrow\}$ $20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$ | M1 | 3.4 |
| | $20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = ...$ | M1 | 1.1b |
| | $\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = ...$ | | |
| | $\{n = \text{awrt } 2.08 \Rightarrow\}$ $d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$ | A1 | 1.1b |
| | **Note:** You can recover the A1 mark for a correct model equation given in part (c) | **(3)** | |
| **(c)** | $d = (0.017)(60)^{2.08}$ | M1 | 3.4 |
| | • $13.333... + 84.918... = 98.251...$ $\Rightarrow$ Sean stops in time | M1 | 3.1b |
| | • $100 - 13.333... = 86.666...$ & $d = 84.918$ $\Rightarrow$ Sean stops in time | A1ft | 3.2a |
| | | **(3)** | |
| | | | **(9 marks)** |

8.

$\frac{dy}{dx} = 3x^2 - 8x$ **[M1, A1]**

Decreasing function $\Rightarrow \frac{dy}{dx} < 0$: $3x^2 - 8x < 0$ **[M1]**

$x(3x - 8) < 0$

The critical values are $x = 0$ and $x = \frac{8}{3}$ **[A1]**

$3x^2 - 8x$ is a concave-up parabola, $< 0$ means below the $x-$axis **[M1] oe**

$0 < x < \frac{8}{3}$ **[A1]**

Set notation: $\left\{x : x > 0\right\} \cap \left\{x : x < \frac{8}{3}\right\}$ **[A1]**

9.

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| (i) | (a) | c – a oe | B1 [1] | 1.2 | | |
| (i) | (b) | a + $\frac{1}{2}$(c – a) or c + $\frac{1}{2}$(a – c)<br>= $\frac{1}{2}$(a + c) or $\frac{1}{2}$a + $\frac{1}{2}$c | M1<br>A1<br>[2] | 3.1a<br>1.1b | a + $\frac{1}{2}$ their (a) or c – $\frac{1}{2}$ their (a)<br>Correct ans without wking: M1A1 | |
| (ii) | | $\overrightarrow{OB}$ = (a + c) | M1 | 3.1a | $\overrightarrow{PB}$ = a + $\frac{1}{2}$(c – a) or a + $\frac{1}{2}$ their (i)(a)<br>or c + $\frac{1}{2}$(a – c)<br>(= $\frac{1}{2}$(a+c) oe), ft their (i)(a)<br>NB $\overrightarrow{PB}$ = $\frac{1}{2}$(a + c) without justification: M0A0A0E0 | or $\overrightarrow{PB}$ = c – $\frac{1}{2}$ their (i)(a) |
| | | $\Rightarrow \overrightarrow{OP} = \frac{1}{2}\overrightarrow{OB}$ Must see previous line | A1* | 1.1 | $\Rightarrow \overrightarrow{PB} = \overrightarrow{OP}$ | or similar with $\overrightarrow{BP}$ or $\overrightarrow{BO}$ |
| | | $\Rightarrow$ *P* is midpt of *OB*<br>or *OPB* is a straight line and *OP* = *PB* | dep* A1 | 2.1 | | |
| | | Hence diagonals of //m bisect one another | E1<br>[4] | 2.2a | dep M1A1A1 | |

10.

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| **12(a)** | Uses model to form one correct equation (PI by *a*=200) (ACF)<br>Accept 1.105*a* + 1.221*b* = 331 | AO3.1b | M1 | $290 = a + b$<br>$331 = ae^{0.1} + be^{0.2}$ |
| | Forms a second correct equation (ACF) | AO1.1a | M1 | $290e^{0.1} = ae^{0.1} + be^{0.1}$<br>$b = \frac{(331 - 290e^{0.1})}{(e^{0.2} - e^{0.1})} = 90.3 = 90$ to the nearest integer |
| | Obtains correct *a* AWRT 200 and *b* AWRT 90 (AG)<br><br>Only award if **both** previous M1's achieved. Do not award marks retrospectively for correct values of *a* and *b* used in part (b) | AO1.1b | A1 | so $a = 200$ |
| **(b)** | Substitutes *t* = 3 and evaluates CAO | AO3.4 | B1 | $200e^{0.3} + 90e^{0.6} = 434$ |

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| **(c)** | Forms inequality (accept < or >) (condone use of equation) FT 'their' value of $a$, but $b$ must be 90 | AO1.1a | M1 | $90e^{0.2t} > 200e^{0.1t}$<br>$e^{0.1t} > \frac{200}{90}$<br>$0.1t > \ln\left(\frac{200}{90}\right)$<br>$t > 10\ln\left(\frac{200}{90}\right) = 7.985$<br>Just less than 8 so during 2024 |
| | Uses logs or calculator to solve 'their' inequality (or equation)<br>If using trial and error must see $t$=7 and $t$=8 tested | AO1.1a | M1 | |
| | Interprets final result. (Do not accept 2025) | AO3.2a | A1 | |
| **(d)** | Gives one limitation of the model.<br>Eg. Model must break down as both $n_A$ and $n_B$ will tend to infinity / model assumes nothing changes / no attempt to control the diseases / all the trees have died / finite number of trees / cure for the disease might be found / other factors such as drought could affect the model / etc. | AO3.5b | E1 | Eventually all of the trees will die so the model will no longer be accurate. |
| | **Total** | | **8** | |