L6 Mock Teacher X 22-23 SOLUTIONS [57]

1

Recalls that correct step is to multiply top and bottom by $2 - \sqrt{n}$ PI by subsequent work	1.2	M1	$\frac{3 - \sqrt{n}}{2 + \sqrt{n}} \times \frac{2 - \sqrt{n}}{2 - \sqrt{n}}$
Multiplies numerator and denominator by ($2 - \sqrt{n}$) to get correct terms (condone sign errors) Does not need to be simplified PI by correct simplification	1.1a	M1	$\frac{6 - 3\sqrt{n} - 2\sqrt{n} + n}{4 + 2\sqrt{n} - 2\sqrt{n} - n}$
Obtains correct simplified numerator and denominator not necessarily in a fraction	1.1b	A1	$\frac{6 + n - 5\sqrt{n}}{4 - n}$
States correct expressions for a and b Or gives expression with a and b correctly identified	1.1b	A1	$a = \frac{6 + n}{4 - n}$ $b = \frac{-5}{4 - n}$
Question 7 Total		**4**	

2.

Critical values of x are $2, -3$ $\{x : x < -3\} \cup \{x : x > 2\}$	**B1** **B1ft** **[2]**	**1.1** **2.5**	Possibly seen in solution using set notation Follow through their two c.v. of x	 $\pm 2, \pm 3$ only Curly brackets only With this notation only

3.

(a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=p-2$	B1	1.2
(ii)	$\log_3\left(\sqrt{x}\right)=\frac{1}{2}p$	B1	1.1b
		(2)	
(b)	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow 2p-4+\frac{3}{2}p=-11\Rightarrow p=...$	M1	1.1b
	$p=-2$	A1	1.1b
	$\log_3 x=-2\Rightarrow x=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
		(4)	
	Alternative for (b) not using (a):		
	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow\log_3\left(\frac{x}{9}\right)^2+\log_3\left(\sqrt{x}\right)^3=-11$ $\Rightarrow\log_3\frac{x^{\frac{7}{2}}}{81}=-11$	M1	1.1b
	$\Rightarrow\frac{x^{\frac{7}{2}}}{81}=3^{-11}$ or equivalent eg $x^{\frac{7}{2}}=3^{-7}$	A1	1.1b
	$x^{\frac{7}{2}}=81\times3^{-11}\Rightarrow x^{\frac{7}{2}}=3^4\times3^{-11}=3^{-7}\Rightarrow x=\left(3^{-7}\right)^{\frac{2}{7}}=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
			(6 marks)

4.

Recalls that the discriminant must be negative, seen anywhere in solution	1.2	B1	For no real solutions the discriminant must be negative
Substitutes 9, 4 and p^2 into the expression b^2-4ac PI	1.1a	M1	$4^2-4\times9\times p^2<0$ $p^2>\frac{4}{9}$
Deduces correct critical values of $\frac{2}{3}$ and $-\frac{2}{3}$	2.2a	A1	
Obtains two correct inequalities for p	2.5	A1	$p>\frac{2}{3}$ or $p<-\frac{2}{3}$
Question 4 Total		**4**	

5.

(a)	$\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ} = 13\mathbf{i} - 15\mathbf{j} - (3\mathbf{i} + 5\mathbf{j})$	M1	1.1a
	$= 10\mathbf{i} - 20\mathbf{j}$	A1	1.1b
		(2)	
(b)	$\left\lvert\overrightarrow{QR}\right\rvert = \sqrt{"10"^2 + "(-20)"^2}$	M1	2.5
	$= 10\sqrt{5}$	A1ft	1.1b
		(2)	
(c)	$\overrightarrow{PS} = \overrightarrow{PQ} + \frac{3}{5}\overrightarrow{QR} = 3\mathbf{i} + 5\mathbf{j} + \frac{3}{5}("10\mathbf{i} - 20\mathbf{j}") = \ldots$ or $\overrightarrow{PS} = \overrightarrow{PR} + \frac{2}{5}\overrightarrow{RQ} = 13\mathbf{i} - 15\mathbf{j} + \frac{2}{5}("-10\mathbf{i} + 20\mathbf{j}") = \ldots$	M1	3.1a
	$= 9\mathbf{i} - 7\mathbf{j}$	A1	1.1b
		(2)	
			(6 marks)

6.

(a)	$(k =)\ 0.8$	B1	1.1b
		(1)	
(b)	$1 = 0.8 + 1.4\mathrm{e}^{-0.5t} \Rightarrow 1.4\mathrm{e}^{-0.5t} = 0.2$	M1	3.1b
	$-0.5t = \ln\left(\frac{0.2}{1.4}\right) \Rightarrow t = \ldots$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\frac{\mathrm{d}P}{\mathrm{d}t} =\right) -0.7\mathrm{e}^{-0.5t}$ $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=2} = -0.7\mathrm{e}^{-0.5\times 2}$	M1	3.1b
	= awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
			(6 marks)

7.

(a)	Rewrites the equation as $y = \sqrt{2}x^{-2}$ PI by correct derivative.	1.1b	B1	$y = \sqrt{2}x^{-2}$ $\frac{dy}{dx} = -\frac{2\sqrt{2}}{x^3}$ Grad at $(2, \frac{\sqrt{2}}{4}) = -\frac{2\sqrt{2}}{8} = -\frac{\sqrt{2}}{4}$ Tangent at $(2, \frac{\sqrt{2}}{4})$ is $y - \frac{\sqrt{2}}{4} = -\frac{\sqrt{2}}{4}(x-2)$ $y = \frac{3\sqrt{2}}{4} - \frac{x\sqrt{2}}{4}$
	Differentiates with their power of x correct provided original power is negative.	1.1a	M1	
	Substitutes $x = 2$ to obtain correct gradient.	1.1b	A1	
	Obtains correct equation of tangent to curve, any form.	1.1b	A1	
	Subtotal		**4**	
(b)	Eliminates y for their tangent and the given curve to find other intersection point. Or Equates $\frac{dy}{dx}$ to the gradient of the perpendicular to their tangent.	3.1a	M1	
	Simplifies to obtain correct cubic equation. PI by $x = -1$ Or Obtains correct equation $-\frac{2\sqrt{2}}{x^3} = 2\sqrt{2}$ (OE)	1.1a	A1	Meets $y = \frac{\sqrt{2}}{x^2}$ when $\frac{\sqrt{2}}{x^2} = \frac{3\sqrt{2}}{4} - \frac{x\sqrt{2}}{4}$ $\frac{1}{x^2} = \frac{3}{4} - \frac{x}{4}$ $x^3 - 3x^2 + 4 = 0$ $(x-2)^2(x+1) = 0$ Other intersection is at $x = -1$ $\frac{dy}{dx} = \frac{-2\sqrt{2}}{(-1)^3} = 2\sqrt{2}$ $2\sqrt{2} \times (-\frac{\sqrt{2}}{4}) = -1$
	Finds other intersection value of $x = -1$	1.1b	A1	
	Substitutes x = -1 to obtain the correct gradient at the other intersection point. Or Obtains $y = \sqrt{2}$ at the other intersection point.	1.1b	B1	

	Completes a reasoned argument to show the required result using the perpendicular gradients condition. Or Completes argument by finding the equation of the line with gradient $-\frac{\sqrt{2}}{4}$ passing through (-1 , $\sqrt{2}$) and verifying that this equation is identical to the equation found in part (a). OE	2.1	R1	Perpendicular to curve so normal
	Subtotal		**5**	

8.

(a)	Differentiates, at least one term correct.	1.1a	M1	$\frac{dy}{dx} = 3x^2 - 6 - \frac{9}{x^2}$ For stationary point $\frac{dy}{dx} = 0$ $3x^2 - 6 - \frac{9}{x^2} = 0$ $3x^4 - 6x^2 - 9 = 0$ $x^4 - 2x^2 - 3 = 0$
	Obtains correct derivative.	1.1b	A1	
	Sets correct derivative = 0 and rearranges to obtain given equation.	2.1	R1	
	Subtotal		**3**	
(b)	Factorises or solves using calculator. PI	1.1a	M1	$(x^2 - 3)(x^2 + 1) = 0$ $(x^2 - 3) = 0$ gives stationary points at $\pm\sqrt{3}$ $(x^2 + 1) = 0$ has no real solutions so there are only two stationary points
	Obtains two correct factors or obtains two correct solutions. ACF	1.1b	A1	
	Concludes that as there are only 2 solutions, there are only 2 stationary points. OE	2.2a	R1	
	Subtotal		**3**	

(c)	Differentiates their $\frac{dy}{dx}$ again, at least one of the two non-zero terms correct, or uses values to test the sign of $\frac{dy}{dx}$ close to their $\pm\sqrt{3}$ OE	1.1a	M1	$\frac{d^2y}{dx^2} = 6x + \frac{18}{x^3}$ At $(\sqrt{3}, 0)$ $\frac{d^2y}{dx^2}$ is positive therefore this is a minimum point At $(-\sqrt{3}, 0)$ $\frac{d^2y}{dx^2}$ is negative therefore this is a maximum point
	Makes consistent deduction about the nature of one of their stationary points from sign of their $\frac{d^2y}{dx^2}$ or the sign of $\frac{dy}{dx}$ close to their $\pm\sqrt{3}$	1.1a	M1	
	States correct coordinates for one stationary point. ACF	1.1b	B1	
	Obtains the correct exact coordinates of both stationary points, along with their correct natures (from correct $\frac{d^2y}{dx^2}$)	1.1b	A1	
	Subtotal		**4**	
(d)	Deduces $y = 0$	2.2a	B1	$y = 0$
	Subtotal		**1**	

9.

5(a)	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) or $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	M1	1.1b
	$p =$ awrt 3.162 or $q =$ awrt 1.072	A1	1.1b
	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) and $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	dM1	3.1a
	$A = 3.162 \times 1.072^t$	A1	3.3
		(4)	
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1	3.4
(b)(ii)	The ratio of algae from one week to the next.	B1	3.4
		(2)	
(c)(i)	5.5 kg	B1	2.2a
(c)(ii)	$4 = "3.162" \times "1.072"^t$ or $\log_{10} 4 = 0.03\,t + 0.5$	M1	3.4
	awrt 3.4 (weeks)	A1	1.1b
		(3)	
(d)	• The model predicts unlimited growth. • The weather may affect the rate of growth	B1	3.5b
		(1)	
			(10 marks)