L6 Mock Teacher X 18-19 SOLUTIONS [73]

1.

3	(i)		5^8	B1 [1]	cao	
	(ii)		$5^{-\frac{1}{4}}$	M1 A1 [2]	Fourth root $\equiv \frac{1}{4}$ soi **cao www**	
	(iii)		$5^{\frac{9}{2}}$	M1 A1 [2]	$(5^{\frac{3}{2}})^3$ or $5^3 \times 5^{\frac{3}{2}}$ or other correct product of two simplified powers of 5 oe **cao www**	

2.

4			Let $y^{\frac{1}{4}} = x$ $2x^2 - 7x + 3 = 0$ $(2x-1)(x-3) = 0$ $x = \frac{1}{2}, x = 3$ $y = \left(\frac{1}{2}\right)^4, y = 3^4$ $y = \frac{1}{16}, y = 81$	M1* M1dep* A1 M1dep* A1 [5]	Use a substitution to obtain a quadratic or factorise into two brackets each containing $y^{\frac{1}{4}}$ Correct method to solve resulting quadratic Both values correct Attempt to raise to the fourth power Correct final answers	**No marks** if whole equation raised to fourth power etc. **No marks** if straight to formula with no evidence of substitution at start and no raising to fourth power/fourth rooting at end. **No marks** if $y^{\frac{1}{4}} = x$ and then $2x - 7x^2 + 3 = 0$. **Spotted solutions:** If **M0 DM0** or **M1 DM0** **SC B1** $y = 81$ **www** **SC B1** $y = \frac{1}{16}$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)
			<u>Alternative by rearrangement and squaring:</u> $2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0$, $7y^{\frac{1}{4}} = 2y^{\frac{1}{2}} + 3$ $49y^{\frac{1}{2}} = 4y + 12y^{\frac{1}{2}} + 9$, $37y^{\frac{1}{2}} = 4y + 9$ $16y^2 - 1297y + 81 = 0$ $(16y-1)(y-81) = 0$ $y = \frac{1}{16}, y = 81$	M2* A1 M1dep* A1	Rearrange and square both sides twice Correct quadratic obtained Correct method to solve resulting quadratic Correct final answers	
			OR methods may be combined: e.g. after $37y^{\frac{1}{2}} = 4y + 9$ $4y - 37y^{\frac{1}{2}} + 9 = 0$ $4x^2 - 37x + 9 = 0$ $(4x-1)(x-9) = 0$ $x = \frac{1}{4}, x = 9$ $y = \left(\frac{1}{4}\right)^2, y = 9^2$	M1* M1dep* A1 M1dep* A1 [5]	Rearrange, square both sides and substitute Correct method to solve resulting quadratic Attempt to square Correct final answers	

3.

1			$\frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{8\sqrt{3}+8}{3-1}$ $4\sqrt{3}+4$	M1 A1 A1 [3]	Multiply top and bottom by $\sqrt{3}+1$ or $-\sqrt{3}-1$ – evidence of multiplying out needed Either numerator or denominator correct Final answer **cao**	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{3}+b$ **A1** Either a or b correct **A1** Both correct

4.

(i)		$3(x^2 - \frac{5}{3}x) + 1$ $3[(x - \frac{5}{6})^2 - \frac{25}{36}] + 1$ $3(x - \frac{5}{6})^2 - \frac{13}{12}$	B1 B1 M1 A1 [4]	$a = 3$ $b = -\frac{5}{6}$ (not $\frac{-\frac{5}{3}}{2}, \frac{-2.5}{3}$) $1 - 3b^2$ or $3 \times (\frac{1}{3} - b^2)$ $c = -\frac{13}{12}$. Allow $-\frac{39}{36}$ etc.	$3(x - \frac{5}{6})^2 + \frac{13}{12}$ **B1 B1 M0 A0** $3(x - \frac{5}{6})\ \ - \frac{13}{12}$ **4/4 BOD** $3(x - \frac{5}{6}x)^2 - \frac{13}{12}$ **B1 B0 M1 A0** $3(x^2 - \frac{5}{6})^2 - \frac{13}{12}$ **B1 B0 M1 A0** $3x(x - \frac{5}{6})^2 - \frac{13}{12}$ **B0 B1 M1 A0** $3(x^2 - \frac{5}{6}) - \frac{13}{12}$ **B1 B0 M1 A0** $3(x + \frac{5}{6})^2 - \frac{13}{12}$ **B1 B0 M1 A0**
(ii)		$(-5)^2 - 4.3.1 = 13$ So 2 real roots	B1 B1ft [2]	ft their discriminant e.g. "$-25 - 12 = -37$ so no roots" scores **B0 B1ft**	Use of $\sqrt{b^2 - 4ac}$ can score **B0 B1**

5. **Differentiates $f(x)$** **[M1]**
$f'(x) = 72 + 30x - 12x^2$ **[A1]**
Attempts to solve $72 + 30x - 12x^2 < 0$ **[M1]**
Roots are $x = 4$ and $x = -1.5$ **[A1]**
Sketch of concave down curve **[M1] oe**
$\{x : x < -1.5\} \cup \{x : x > 4\}$ **[A1] oe**

6.

8	**(a)**		$\log 2^{n-3} = \log 18000$	M1*	Introduce logs and drop power	Can use logs to any base, as long as consistent on both sides, and allow no explicit base as well If taking $\log_2$ then base must be explicit Allow M1 for $n - 3 \ \log 2 = \log 18000$
			$(n - 3) \log 2 = \log 18000$	A1	Obtain $(n - 3) \log 2 = \log 18000$ or equiv	Or $n - 3 = \log_2 18000$ Brackets now need to be seen explicitly, or implied by later working
			$n - 3 = 14.1$	M1d*	Attempt to solve for n	Correct order of operations, and correct operations ie M0 for $\log_2 18000 - 3$ M0 if logs used incorrectly eg $n - 3 = \log(^{18000}/_2)$
			$n = 17.1$	A1	Obtain 17.1, or better	Final answer must be correct for all sig fig shown ($n = 17.13570929...$) 0/4 for answer only, or T&I If rewriting eqn as $2^{n-3} = 2^{14.1}$ then 0/4 unless evidence of use of logs to find the index of 14.1
				[4]		
	(b)		$2\log_2 x - \log_2 y = 7$	M1	Correct use of one log law - on a correct equation	Either on first eqn to get $\log_2(xy) = 8$, or on second eqn to get at least $\log_2 x^2 - \log_2 y = 7$ Allow for one correct use, even if error made with other equation Must be used on a correct equation so M0 if an error has already occurred eg $\log(x^2/y) = 2\log(xy) = 2(\log x + \log y)$ is M0
			$(\log_2 x + \log_2 y) + (2\log_2 x - \log_2 y) = 15$	M1	Attempt to eliminate one variable	To get an equation in just one variable, which may or may not still involve logs Must be a sound algebraic process with the two equations that they are using, though errors may have been made earlier with log / index laws
			$3\log_2 x = 15$	A1	Obtain correct equation in just one variable	Which may or may not still involve logs Depending on the method used, possible equations are $3\log_2 x = 15$, $\log_2 x^3 = 15$, $x^3 = 32768$ or $3\log_2 y = 9$, $\log_2 y^3 = 9$, $y^3 = 512$ The variable should only appear once so $\log_2 x^2 + \log_2 x = 15$ is A0 until the two log terms are correctly combined
			$x = 2^5$	M1	Correctly use 2^k as inverse of $\log_2$	At any stage - may even be the very first step to obtain $x^2/y = 128$ M0 for eg $\log_2 x + \log_2 y = 8$ becoming $x + y = 2^8$ as incorrect method to remove logs
			$x = 32, y = 8$	A1	Obtain $x = 32, y = 8$	Both values required, and no others Answer only, with no evidence of log or index work, is 0/5
				[5]		

7. **When $x = 4, y = 5$** **[B1]**

Function split into two fractions: $\frac{x^2}{\sqrt{x}} - \frac{6}{\sqrt{x}}$ **[M1] oe**

Simplifies to $x^{\frac{3}{2}} - 6x^{-\frac{1}{2}}$ **[A1]**

Differentiates $\frac{3}{2}\sqrt{x} + 3x^{-\frac{3}{2}}$ **[M1] Allow one error**

Substitute $x = 4$ into *their* equation for $\frac{dy}{dx}$ **[M1]** $\frac{dy}{dx} = 3\frac{3}{8} = \frac{27}{8}$

Attempts to find equation of tangent **[M1]**

$y = \frac{27}{8}x - \frac{17}{2}$ **[A1]**

8.

5	(a)		Differentiate to produce $ke^{-0.33t}$ Obtain $-19.14e^{-0.33t}$ or $19.14e^{-0.33t}$ Obtain -5.1 or 5.1	M1 A1 A1 [3]	where constant k is different from 58 or unsimplified equiv whatever they claim value represents; accept 5.11 but not greater accuracy	method must involve differentiation
5	(b)		Either: State or imply formula $42e^{kt}$ or $42a^t$ Attempt to find k from $42e^{6k} = 51.8$ or a from $42a^6 = 51.8$ Obtain $k = 0.035$ or $a = 1.0356$ Substitute 24 to obtain value between 97.1 and 97.3 inclusive	B1 M1 A1 A1	$42e^{-kt}$, $42e^{-kx}$, etc. also acceptable using sound process involving logarithms at least as far as $6k = \ldots$ or $a = \ldots$ or greater accuracy 0.03495… or exact equiv $\frac{1}{6}\ln\frac{37}{30}$ allow greater accuracy than 3 s.f.	
			Or: Use ratio $\frac{51.8}{42}$ in calculation Attempt calculation of form $42 \cdot r^n$ Obtain $42 \cdot (\frac{51.8}{42})^4$ or $51.8 \cdot (\frac{51.8}{42})^3$ Obtain value between 97.1 and 97.3 inclusive	B1 M1 A1 A1 [4]	allow greater accuracy than 3 s.f.	

9. **Volume,** $V = x(x-3)(x-7) = x^3 - 10x^2 + 21x$ **[M1]**

Differentiates their volume equation $\frac{dV}{dx} = 3x^2 - 20x + 21$ **[M1]**

Attempts to solve $\frac{dV}{dx} = 0$ **[M1]**

$x = 1.31$ ***and*** $x = 5.36$ **[A1]**

Attempts to prove maximum/minimum via gradient change or 2nd derivative **[M1]**

$x = 1.31$ **is a maximum** **[A1]**

10. (a) (i) 2**b** − 2**a** **[A1]**

(ii) 2**c** − 2**b** **[A1]**

(iii) **b** **[A1]**

(iv) $\overrightarrow{SR} = \overrightarrow{SC} + \frac{1}{2}\overrightarrow{CB}$ **[M1] oe**

b **[A1]**

(b) **Parallel and equal** **[A1]**

(c) **Parallelogram** **[A1]**

11. (a) $\ln P = \ln Ae^{kt}$ **[M1] oe**

$\ln P = kt + \ln A$ **[A1]**

[2]

(b) **Horizontal axis labelled** t **[B1]**

Vertical axis labelled $\ln P$ **[B1]**

(c) $y - \text{intercept} = 2.89 = \ln A$ **[M1]**

$A = e^{2.89} = 18$ **[A1]**

$\text{Gradient} = \frac{4.44-2.89}{5} = 5$ **[M1]**

$k = 31$ **[A1]**

[4]

(d) $18\, e^{0.31t} = 1000$

$e^{0.31t} = 55.555\ldots$ **[M1] oe**

$0.31t = \ln 55.555\ldots$ **[M1] oe**

$t = 12.959\ldots = 13$ **days** **[A1]**

Overestimate because:

...people may get better

...ill people may be quarantined [R1] oe 1 reason required

[4]