

# L6 Mathematics Mock

## Paper 1 (Teacher X)

### January 2020

### 2019-2020

**Duration: 1 hour 30 minutes**

**Total number of marks: 73**

*Write your answers on file paper provided.*

**You are permitted to use a scientific or graphical calculator in this paper.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

**Relevant information from the formula booklet is given below:**

#### Formulae

AS Level Mathematics A (H230)

#### Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1}b + {}^n C_2 a^{n-2}b^2 + \dots + {}^n C_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

1.

Find, using algebra, all real solutions to the equation

(i)  $16a^2 = 2\sqrt{a}$  (4)

(ii)  $b^4 + 7b^2 - 18 = 0$  (4)

2.

Given that the point  $A$  has position vector  $4\mathbf{i} - 5\mathbf{j}$  and the point  $B$  has position vector  $-5\mathbf{i} - 2\mathbf{j}$ ,

(a) find the vector  $\vec{AB}$ , (2)

(b) find  $|\vec{AB}|$ .

Give your answer as a simplified surd.

(2)

3.

Show that  $\frac{\sqrt{6}}{\sqrt{3} - \sqrt{2}}$  can be expressed in the form  $m\sqrt{n} + n\sqrt{m}$ , where  $m$  and  $n$  are integers.

Fully justify your answer.

[4 marks]

4.

Show that, for  $x > 0$

$$\log_{10} \frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3 \equiv 2(-1 + \log_{10} 3x)$$

[4 marks]

5.

Express the following in the form  $2^p$ .

(i)  $(2^5 \div 2^7)^3$  [2]

(ii)  $5 \times 4^{\frac{2}{3}} + 3 \times 16^{\frac{1}{3}}$  [3]

6.

A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \quad x > 0$$

(a) Find, in simplest form,  $\frac{dy}{dx}$  (3)

(b) Hence find the exact range of values of  $x$  for which the curve is increasing. (2)

7.

(i) Express  $4 + 12x - 2x^2$  in the form  $a(x + b)^2 + c$ . [4]

(ii) State the coordinates of the maximum point of the curve  $y = 4 + 12x - 2x^2$ . [2]

8.

A curve has equation  $y = 2x^2$ . The points  $A$  and  $B$  lie on the curve and have  $x$ -coordinates 5 and  $5 + h$  respectively, where  $h > 0$ .

(i) Show that the gradient of the line  $AB$  is  $20 + 2h$ . [3]

(ii) Explain how the answer to part (i) relates to the gradient of the curve at  $A$ . [1]

(iii) The normal to the curve at  $A$  meets the  $y$ -axis at the point  $C$ . Find the  $y$ -coordinate of  $C$ . [3]

9.

The value of a car,  $\pounds V$ , can be modelled by the equation

$$V = 15\,700e^{-0.25t} + 2300 \quad t \in \mathbb{R}, t \geq 0$$

where the age of the car is  $t$  years.

Using the model,

(a) find the initial value of the car. (1)

Given the model predicts that the value of the car is decreasing at a rate of  $\pounds 500$  per year at the instant when  $t = T$ ,

(b) (i) show that

$$3925e^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)* (6)

The model predicts that the value of the car approaches, but does not fall below,  $\pounds A$ .

(c) State the value of  $A$ . (1)

(d) State a limitation of this model. (1)

10.

Prove that the curve with equation

$$y = 2x^5 + 5x^4 + 10x^3 - 8$$

has **only one** stationary point, stating its coordinates.

[6 marks]

11.

Find the set of values of  $k$  for which the equation  $x^2 + 2x + 11 = k(2x - 1)$  has two distinct real roots. [7]

12.

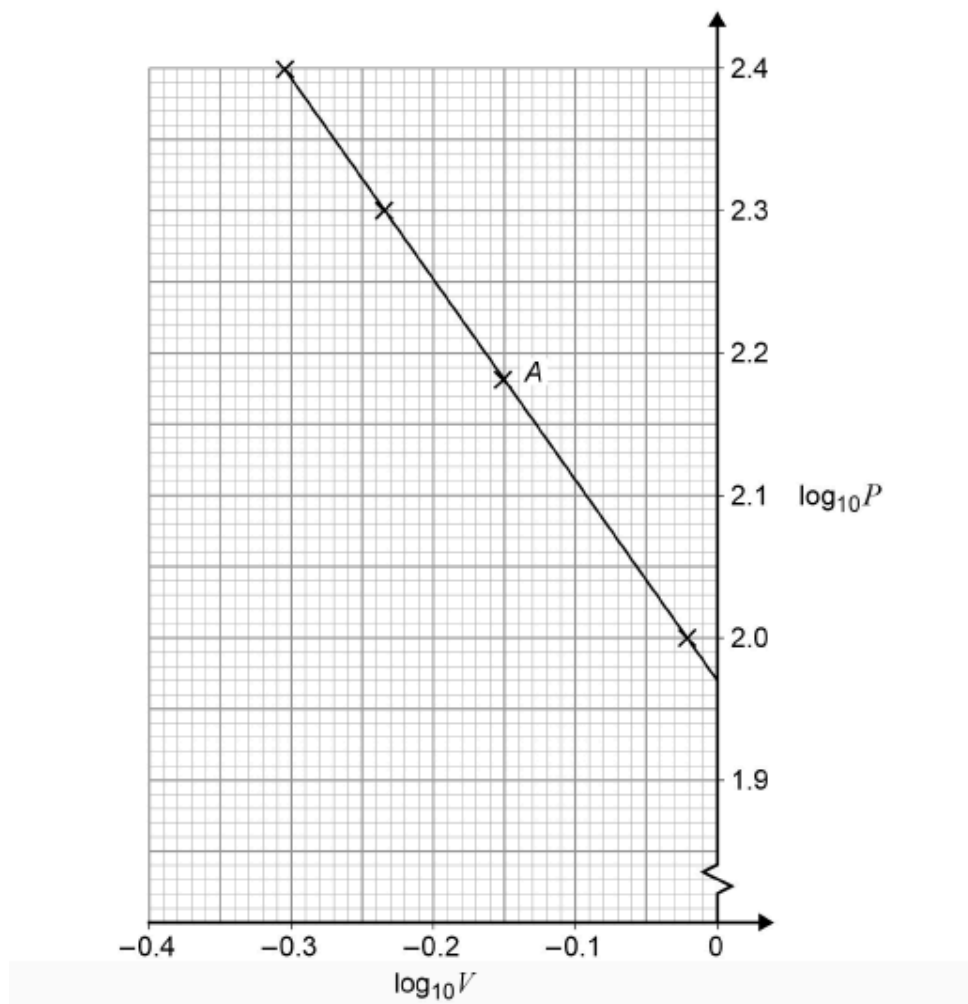
Maxine measures the pressure,  $P$  kilopascals, and the volume,  $V$  litres, in a fixed quantity of gas.

Maxine believes that the pressure and volume are connected by the equation

$$P = cV^d$$

where  $c$  and  $d$  are constants.

Using four experimental results, Maxine plots  $\log_{10}P$  against  $\log_{10}V$ , as shown in the graph below.



- (a) Find the value of  $P$  and the value of  $V$  for the data point labelled  $A$  on the graph. [2 marks]
- (b) Calculate the value of each of the constants  $c$  and  $d$ . [4 marks]
- (c) Estimate the pressure of the gas when the volume is 2 litres. [2 marks]