Calculating binomial coefficients

Starter

1. *(Review of last lesson)* Find the terms indicated in the expansions of the following:

 (a) $(4+2p^2)^6$ term in p^6 (b) $\left(4p+\dfrac{2}{p}\right)^5$ term in p^3

2. *(Review of last lesson)* Expand $(1+x+6x^2)^3$.

3. Factorial notation, $n!$, is given by:
$$n! = n \times (n-1) \times (n-3) \times \ldots \times 3 \times 2 \times 1$$
with $0!$ defined as 1.
State the values of: (a) $4!$ (b) $7!$ (c) $12!$

4. $^nC_r = \dfrac{n!}{(n-r)!r!}$
 (a) Show that $^nC_0 = 1$ and $^nC_n = 1$.
 (b) Find expressions in terms of n, for:
 (i) $^nC_1$ (ii) $^nC_{n-1}$ (iii) $^nC_2$ (iv) $^nC_{n-r}$

Notes

Key notation and facts

Factorial: $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$
$0! = 1$

Choose notation: $^nC_r \equiv {}_nC_r \equiv \dbinom{n}{r}$

$^nC_r = \dfrac{n!}{(n-r)!r!}$

$^nC_0 = {}^nC_n = 1$

$^nC_1 = {}^nC_{n-1} = n$

$^nC_{n-r} = {}^nC_r$

E.g. 1 Without a calculator, find the value of:
(a) $^5C_2$ (b) $^7C_3$ (c) $^8C_6$ (d) $^9C_3$

Working: (a) $^5C_2 = \dfrac{5!}{(5-2)!2!} = \dfrac{5!}{3!2!} = \dfrac{5 \times 4}{2 \times 1} = 10$

Calculating unknowns in an expansion

The general term in the expansion of $(a+b)^n$ is: $^nC_r a^{n-r} b^r$

E.g. 2 In the expansion of $(1+ax)^4$, the coefficient of x^3 is 1372. Find the value of a.

E.g. 3 Given that the expansion of $(1+ax)^n$ begins $1+36x+576x^2$, find the values of a and n.

Video: Binomial expansion

Exam questions: Binomial expansion (positive integer powers)

Exam questions: Comparing coefficients

Solutions to Starter and E.g.s

Exercise

p155 9B Qu 1i, 2i, 3-4, (5-6 red)

Summary

Factorial: $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$

$0! = 1$

Choose notation: ${}^nC_r \equiv {}_nC_r \equiv \binom{n}{r}$

${}^nC_r = \dfrac{n!}{(n-r)!r!}$

${}^nC_0 = {}^nC_n = 1$

${}^nC_1 = {}^nC_{n-1} = n$

${}^nC_{n-r} = {}^nC_r$

The general term in the expansion of $(a+b)^n$ is: ${}^nC_r a^{n-r} b^r$