Connected Particles (vertical)

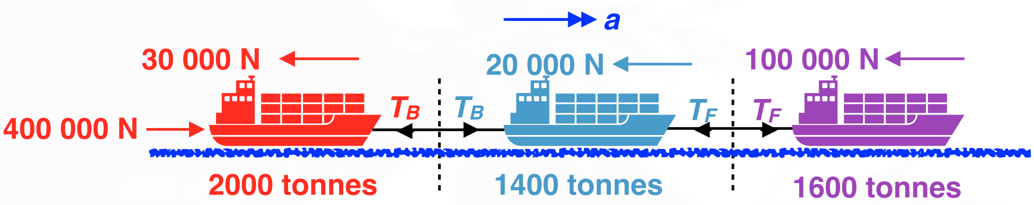
Starter

1. (Review of last lesson) Three barges travel down a river in line. Only the rear barge has an engine, which produces a forward force of $400k$ N. The masses of the barges are 1600 tonnes, 1400 tonnes and 2000 tonnes and the water exerts resistance of $100k$ N, $20k$ N and $30k$ N respectively (front to back). Find:
   (a) the acceleration of the barges

   (b) the forces in the couplings joining the barges.

N.B. $400k$ N $= 400000$ N

Working: **The diagram for the question is below (the nearest to a barge I could find is a cargo ship — I hope you're not too upset by that).**
Since the driving force is coming from the barge at the back the directions of the tensions in the couplings are reversed. It wouldn't matter if you got them the wrong way round as the values would come out negative.
Let T_F be the tension in the coupling between the front 2 barges and T_B be the tension in the coupling between the back 2 barges.
Remember: 1 tonne $\equiv 1000$ kg



Notes

Last lesson we looked at a car pulling a caravan i.e. two connected particles that moved horizontally.

This lesson we will consider connected particles that move vertically. This will include a rope or string passed over a pulley or smooth peg.

Modelling assumptions

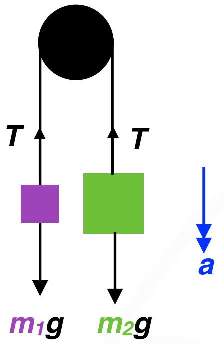
- The axis of the pulley or the peg is smooth i.e. no frictional force
- The pulley or peg is light - the mass of the pulley can be ignored

These mean that the tension on both sides of the string is equal.

Take each part of the string separately, using T for tension. We consider each mass on its own.

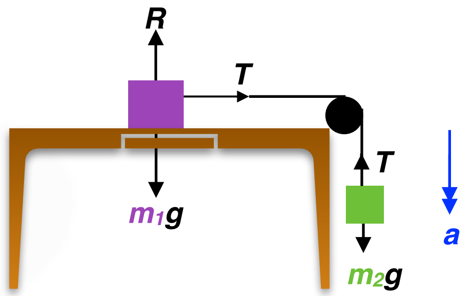
Basic pulley

The string passes over the pulley and has two masses, m_1 and m_2, where $m_1 < m_2$, at the ends of the strings



Pulley at edge of table

A mass m_1 lies on a smooth table and is attached to mass m_2, which is free to hang downwards, by means of a pulley.



The tension is each part of the string is equal.
While the string remains taut, the acceleration of both particles is the same
We use $F = ma$ separately for each mass.

Basic pulley:

$F = ma(\uparrow)\ m_1$: $\quad T - m_1g = m_1a$

$F = ma(\downarrow)\ m_2$: $\quad m_2g - T = m_2a$

Adding eliminates T:

$$m_2g - m_1g = m_1a + m_2a$$

$$g(m_2 - m_1) = a(m_1 + m_2)$$

So

$$a = \frac{g(m_2 - m_1)}{m_1 + m_2}$$

Pulley at edge of table:

$F = ma(\rightarrow)\ m_1$: $\quad T = m_1a$

$F = ma(\downarrow)\ m_2$: $\quad m_2g - T = m_2a$

Replacing T by m_1a in the 2nd equation:

$$m_2g - m_1g = m_2a$$

$$g(m_2 - m_1) = m_2a$$

So

$$a = \frac{g(m_2 - m_1)}{m_2}$$

N.B. For the mass on the table, m_1, only the force T acts horizontally so its weight, m_1, and the normal contact force, R, are not required.

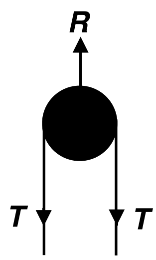
N.B. Provided the string remains taut
- The tensions in different parts of the same string are equal.
- The acceleration of the two connected particles are equal.

Force exerted on the pulley by the string

The string exerts a force on the pulley and their directions are reversed compared to the tension from the point of view of the masses.

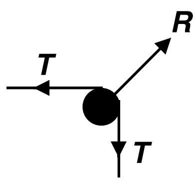
Basic pulley: the tensions in the string act vertically downwards so we can resolve vertically.

$R(\uparrow)$: $\quad R = 2T$



Edge of table: the tensions act at right angles so the force at the pulley acts at 45^{o} to the horizontal. To find R we resolve in its direction.

$$R = T\cos 45 + T\cos 45$$
$$R = 2T\cos 45$$



E.g. 1 A light inextensible string passes over a smooth fixed pulley and carries particles of masses 5 kg and 7 kg, one at each end. If the system is moving freely, find in terms of g:

(a) the acceleration of each particle
(b) the tension in the string
(c) the force exerted on the pulley by the string.

Make sure you draw a diagram

E.g. 2 A small block of mass 6 kg rests on a table top and is connected by a light inextensible string that passes over a smooth pulley, fixed on the edge of the table, to another small block of mass 5 kg which is hanging freely. Find, in terms of g, the acceleration of the system and the tension in the string if:

(a) the table is smooth
(b) the table is rough and exerts a frictional force of $2g$ N.

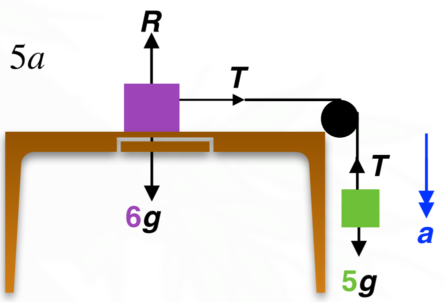
Make sure you draw a diagram.

Working (a) $F = ma(\rightarrow)$ 6 kg mass: $T = 6a$

$F = ma(\downarrow)$ 5 kg mass: $5g - T = 5a$

Add the equations to eliminate T:

$$5g = 11a$$
$$a = \frac{5g}{11} \text{ m/s}^2$$



Replace a by $\frac{5g}{11}$ in the 1st (or 2nd) equation:

$$T = 6 \times \frac{5g}{11}$$
$$T = \frac{30g}{11} \text{ N}$$

The the acceleration of the system is $\frac{5g}{11}$ m/s² and the tension in the string is $\frac{30g}{11}$ N.

Continued motion

The situation may arise where the string breaks or one of the masses hits the ground. In such cases, the mass that is going up will continue to rise for a short time and it moves as if a particle in ***vertical motion under the effect of gravity with*** $a = -9.8$, i.e. use the ***SUVAT*** equations.

E.g. 3 Two particles P and Q are connected by a long inextensible string passing over a smooth pulley. The mass of P is m, the mass of Q is $2m$ and the particles are held so that each is 3 metres below the pulley. The system is released from rest and after 1 second the string breaks. Find:

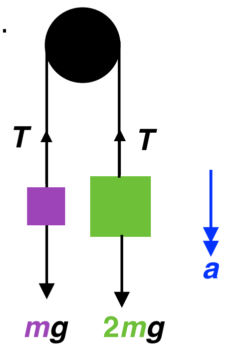
(a) the speed of each particle at that instant

(b) the further distance that P rises.

Working (a) Consider what we know from a SUVAT perspective.

$u = 0, t = 1, v = ?$

To find the final speed, we need to calculate the acceleration of the system.



(b) ***After the string breaks, particle*** P ***moves under the influence of gravity so its acceleration is*** -9.8.

Its initial velocity is the final velocity found in (a)

At its highest point, its velocity is 0.

Video: Pulley - vertical strings (1st video)

Video: Pulleys - vertical strings (continued motion - 2nd video)

Video: Pulley - edge of table

Video (force on pulley): Resultant force on a pulley

Pulley - vertical strings EQ

Pulley - edge of table EQ

Solutions to Starter and E.g.s

Exercise

p527 22E Qu 1-8

Summary

Assumptions

- The axis of the pulley, or the peg, is smooth i.e. no frictional force
- The pulley or peg is light — the mass of the pulley can be ignored

These mean that the ***tension on both sides of the string is equal***.

Provided the string remains taut ***the acceleration of the two connected particles is equal***.

Force exerted on the pulley by the string:

Basic pulley: $R = 2T$

Edge of table: $R = 2T\cos 45$

Continued motion — particles move ***vertically under the effect of gravity with*** $a = -9.8$, i.e. use the ***SUVAT*** equations.