Cosine rule

Starter

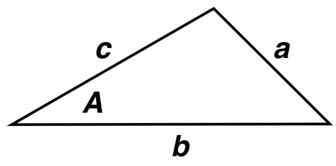
1. *(Review of last lesson)* The triangle ABC has $AB = 4.6$ cm, $CA = 7.1$ cm and $\angle ACB = 33°$. Find the value(s) of $\angle ABC$ and the corresponding length(s) of BC.

Notes

The cosine rule was met in the GCSE course.

Information needed to use the cosine rule

What information do we need to find a side/an angle using the cosine rule?



To find a ***side***: need ***two sides and the angle between them***

Formula: $a^2 = b^2 + c^2 - 2bc \cos A$

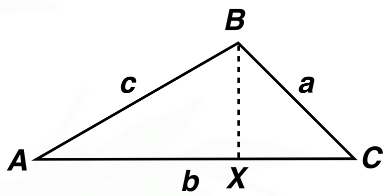
To find an ***angle***: need ***all 3 sides***

Formula: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

N.B. "$-a^2$" comes from the side opposite the angle we are trying to find.

N.B. Always draw a diagram.

Proof of the cosine rule



Let BX be perpendicular to AC so that $b = AX + CX$

N.B. $b = AC$

We want to find a formula for side a in terms of b, c and A.
To do this, we will use Pythagoras in the $\triangle BCX$.
Before this we have to find CX and BX in terms of b, c and A

Finding CX**:** Using trigonometry, $\cos A = \dfrac{AX}{c} \quad \Rightarrow \quad AX = c \cos A$

$CX = AC - AX = b - c \cos A$ ***an expression for*** CX ***is ready***

Finding BX**:** Using Pythagoras, $BX^2 = AB^2 - AX^2$

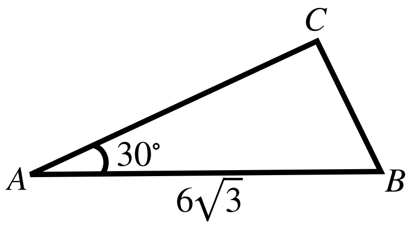
$= c^2 - (c \cos A)^2$ ***an expression for*** BX ***is ready***

Finding a**:** Use Pythagoras in $\triangle BCX$:

$$BC^2 = BX^2 + CX^2$$
$$a^2 = c^2 - (c \cos A)^2 + (b - c \cos A)^2$$
$$a^2 = c^2 - c^2 \cos^2 A + b^2 - 2bc \cos A + c^2 \cos^2 A$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

— formula for cosine rule

E.g. 1 In the triangle shown the side AC is twice the length of BC. Calculate the length of BC, giving your answer exactly.



E.g. 2 A town clock has hands that are of lengths 86 cm and 115 cm. Find the times of the day when the minute hand is pointing to the number 6 and the distance between the tips of the hands is 186 cm.

An arrow is ***only*** drawn between ***known angles and their opposite sides***.

Arrow $\Rightarrow$ use the ***sine rule***

No arrow $\Rightarrow$ use the ***cosine rule***

	Side	Angle
Sine rule	Need a known angle opposite a known side $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$	Need a known angle opposite a known side $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$
Cosine rule	Need two side and the angle between $a^2 = b^2 + c^2 - 2bc \cos A$	Need all 3 sides: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

N.B. Always draw a diagram.

E.g. 3 Ayton is directly north of Byford. A third town, Canfield is 9.93 km from Ayton on a bearing of $128°$. The distance from Byford to Canfield is 16.49 km. Find the bearing of Canfield from Byford.

E.g. 4 A boat is sailing directly towards a cliff. The angle of elevation of a point on top of the cliff and straight ahead of the boat increases from $10°$ to $15°$ as the ship sails a distance of 50 m. Calculate the height of the cliff, giving your answer to the nearest centimetre.

Video: Cosine rule

Solutions to Starter and E.g.s

Exercise

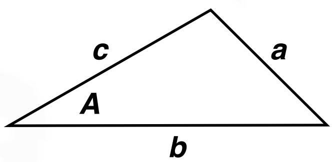
p212 11B Qu1i, 2i, 3-8, (9 red)

Summary

Information needed to use the cosine rule:

To find a ***side***: need ***two sides and the angle between them***

Formula: $a^2 = b^2 + c^2 - 2bc \cos A$



To find an ***angle***: need ***all 3 sides***

Formula: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

N.B. "$-a^2$" comes from the side opposite the angle we are trying to find.

When to use the sine or cosine rule:

An arrow is ***only*** drawn between ***known angles and their opposite sides***.

Arrow $\Rightarrow$ use the ***sine rule***

No arrow $\Rightarrow$ use the ***cosine rule***