Critical region for a hypothesis test

Starter

1. *(Review of last lesson)* Research suggests that Kitty Food is eaten by 6 out of 10 cats in the UK. Following an advertising campaign, company executive want to know if the proportion has changed. In a random sample of 147 cats, it was found that 99 ate their brand of cat food.
   (a) Test at the $5\ \%$ significance level whether the cat food company's claim is correct.
   (b) Why might the company executives not be happy with the outcome? What would be the cheapest way to change the situation?

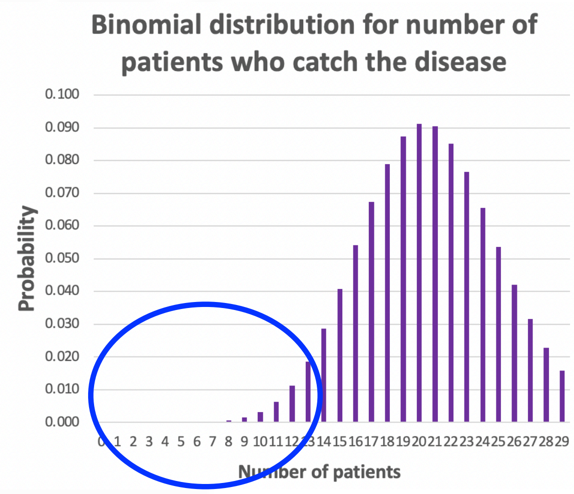
2. Historically a pharmaceutical company has found that $8.3\ \%$ of people catch a certain disease after receiving their vaccine. Biologists from the company develop a new vaccine which they believe reduces the proportion of patients who catch the disease. They test the vaccine on 250 patients and monitor them to see if they catch the disease. What is the maximum number of patients who can catch the disease but still show that the vaccine is an improvement, based on a $5\ \%$ significance level?
   Hint: use trial and improvement with your calculator.

Notes

The answer to question 2 of the Starter is called the ***critical value*** and $X \leq 13$ is called the ***critical region***.

The set of values for which H_0 ***is rejected***, i.e. the values which are ***significant***, is called the ***critical region***. The ***critical value*** is the first value that lies in the critical region, or ***values*** if it is a two-tailed test. They can be found using the ***BinomialCD*** function of a calculator.

One-tailed tests have ***one critical region***, while ***two-tailed tests*** have ***two critical regions*** each less than or equal to the significance level.



While $\alpha\ \%$ is the significance level of the hypothesis test, the sum of the probabilities within the critical region is the ***actual significance level*** of the test.

For example, in question 2 of the Starter, the critical value is 13 so the ***actual significance level*** of the test is $P(X \leq 13) = 0.0417$ or $4.17\ \%$ This value represents the probability that H_0 will be rejected when it is actually true i.e. the probability of a ***Type I error***.

Type I error (or ***false positive***)— rejecting the null hypothesis when it is actually true.
Type II error (or ***false negative***) — not rejecting the null hypothesis when it is actually false.

For a two-tailed test, the ***actual significance level*** is the sum of the probabilities in the two critical regions.

E.g. 1 Researchers believe that the children of farmers are less likely to catch the common cold than other children. Medical records suggest that $73\ \%$ of children catch a cold during the month of December. The researchers monitor the health of 28 randomly chosen farmers' children during one December.

(a) State the critical region at the $3\ \%$ level.

(b) Find the actual significance level.

(c) In fact, 13 of the 28 farmers' children become ill. State the researchers conclusions.

Working: (a) "children of farmers are less likely" $\Rightarrow$ one-tailed test

$H_0 : p = 0.73$ where p is the proportion of children who catch a cold during December.

$H_1 : p < 0.73$

Under H_0, $X \sim$ B $(28, 0.73)$.

$\alpha = 0.02$

$P(X \leq 16) = 0.0512 \nless 0.03$

$P(X \leq 15) = 0.0215 < 0.03$

So less than or equal to 15 farmers' children who get ill is the critical region at the $3\ \%$ level.

(b) $P(X \leq 15) = 0.0215$

The actual significance level is $2.15\ \%$

(c) Since $13 \leq 15$, researchers would reject H_0. There is sufficient evidence to suggest that farmers' children catch the common cold less than non-farmer's children.

E.g. 2 A mathematics department changes the way it teaches GCSE in order to encourage more students to take the subject at A level, which, historically is at $38\ \%$. Over two academic years a random sample of 72 students is chosen.

(a) State the critical region at the $5\ \%$ level.

(b) Find the probability of rejecting the null hypothesis when it is actually true.

(c) From the random sample, 32 students took mathematics at A level. Decide whether more students have taken A level maths.

E.g. 3 A company that manufactures microwaves upgrades its factory and wants to know whether the proportion of faulty microwaves has changed. Before the upgrade the $16.3\ \%$ of microwaves were faulty. The technicians take a sample of 40 microwaves.

(a) Find the critical regions for the test at the $5\ \%$ level.

(b) What is the probability of getting a false positive result?

(c) In fact, the number of faulty microwaves in the sample was 11. State whether this value is significant, giving a reason for your answer.

Video: Critical values - lower tail tests
Video: Critical values - upper tail tests

Exam questions: Hypothesis tests with Binomial distribution

Solutions to Starter and E.g.s

Exercise
p406 18C Qu 1i, 2i, 3-7, (8 red)

Summary
The set of values for which H_0 ***is rejected***, i.e. the values which are ***significant***, is called the ***critical region***. The ***critical value*** is the first value that lies in the critical region, or ***values*** if it is a two-tailed test.

One-tailed tests have ***one critical region***, while ***two-tailed tests*** have ***two critical regions*** each less than or equal to the significance level.

While $\alpha\ \%$ is the significance level of the hypothesis test, the sum of the probabilities within the critical region(s) is the ***actual significance level*** of the test.

Type I error (or ***false positive***)— rejecting the null hypothesis when it is actually true.
Type II error (or ***false negative***) — not rejecting the null hypothesis when it is actually false.