Definite integration

Starter

1. **(Review of last lesson)**

   Given that $\frac{dy}{dx} = 2 + 3x^2$ and that $x = 2$ when $y = 9$, find y as a function of x.

2. **(Review of last lesson)** A curve has a gradient function of $2x + k$, where k is a constant. It crosses the $y-$axis at $(0, -3)$ and the $x-$axis at $(1, 0)$ and $(a, 0)$. Find the equation of the curve and the value of a.

Notes

The integration looked at so far is called ***indefinite integration***. When numbers, or ***limits***, are included on the integration symbol, it becomes ***definite integration***, e.g. $\int_1^3 2x\,dx$.

N.B. The numbers on the integration symbol are called limits.

Indefinite integration	**Definite integration**
$\int 2x\,dx = x^2 + c$	$\int_1^3 2x\,dx = \left[x^2\right]_1^3 = 3^2 - 1^2 = 8$
No limits on the integration symbol. Includes the constant of integration, $+c$. The answer is a function of x.	Limits on the integration symbol The constant of integration does not appear. The answer is a number.

What happens to "$+c$" under definite integration?

Let's imagine include the constant of integration:

$$\int_1^3 2x\,dx = \left[x^2 + c\right]_1^3 = (3^2 + c) - (1^2 + c) = 8$$

The c's will always cancel each other out and therefore we ***do not write $+c$ with definite integration***.

Important results

- Multiplying an integral by a scalar: $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

E.g. $\int_2^9 5x\,dx = 5\int_2^9 x\,dx$ ***don't take a scalar out if integration becomes harder***

- Reversing the limits: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

E.g. $\int_1^5 -x^4\,dx = \int_5^1 x^4\,dx$ ***useful when the function is negative***

- Splitting an integral into two parts: $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

E.g. $\int_1^5 x^2 dx = \int_1^4 x^2 dx + \int_4^5 x^2 dx$ $f(x)$ **must be continuous — no asymptotes**

- Two functions: $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

E.g. $\int_3^7 (x^3 + 3x)dx = \int_3^7 x^3 dx + \int_3^7 3x dx$ ***usually not needed***

E.g. 1 Find the exact value of: (a) $\int_{-1}^{2} (6x^2 + 5)dx$ (b) $\int_3^4 (x^2 + 3x)dx$

(c) $\int_4^9 \sqrt{x} dx$ (d) $\int_3^7 dx$

Working: (a) $\int_{-1}^{2} (6x^2 + 5)dx = \left[2x^3 + 5x\right]_{-1}^{2}$

$= \left(2 \times 2^3 + 5 \times 2\right) - \left(2 \times (-1)^3 + 5 \times (-1)\right)$

$= 33$

E.g. 2 Given that $\int_0^a x^3 dx = 64$, find a, where $a > 0$.

E.g. 3 Let $\int_a^b f(x)dx = 15$ and $\int_a^b g(x)dx = -7$. Find $\int_a^b (3f(x) - 4g(x))dx$.

E.g. 4 Find the possible values of A that satisfy $\int_2^3 (1 - 2Ax)dx = 6A^2$.

Video: Definite integration

Definite integration EQ

Solutions to Starter and E.g.s

Exercise
p301 15D Qu 1iace…, 2-9

Summary
When numbers, or ***limits***, are included on the integration symbol, it becomes ***definite integration***,
We do not write $+c$ with definite integration.

Multiplying an integral by a scalar: $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

Reversing the limits: $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Splitting an integral into two parts: $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

Two functions: $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$