Deriving the Constant Acceleration Formulae

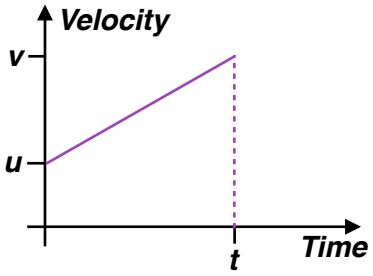
Starter

When a body moves with constant acceleration, the following letters are used:

s — displacement
u — initial velocity
v — final velocity
a — acceleration
t — time

Hence the following equations of motion are sometimes called the $SUVAT$ equations.

1. Consider the velocity-time graph.



(a) Using the fact that the gradient of a line is the acceleration, find an equation involving u, v, a and t.
Rearrange your equation to make v the subject.

(b) Using the fact that the area under the line is the displacement, find an equation involving s, u, v and t.

Working:

(a) Gradient $\equiv a = \dfrac{v-u}{t}$

Rearranging gives: $v = u + at$

(b) Area under the line $\equiv s = \dfrac{1}{2}(u+v)t$ ***area of trapezium***

Notes

From the starter there are 2 equations:

$v = u + at$ does not contain s

$s = \dfrac{1}{2}(u+v)t$ does not contain a

A further 3 equations can be derived by replacing a letter in $s = \dfrac{1}{2}(u+v)t$ by an expression from $v = u + at$.

Replacing v

Replace v by $u + at$ in $s = \dfrac{1}{2}(u+v)t$:

$$s = \frac{1}{2}(u+u+at)t$$
$$s = \frac{1}{2}(2ut + at^2)$$
$$s = ut + \frac{1}{2}at^2$$ No v

Replacing u

Rearrange $v = u + at$: $u = v - at$

Replace u by $v - at$ in $s = \dfrac{1}{2}(u+v)t$:

Replacing t
Rearrange $v = u + at$:

Video: Deriving the constant acceleration (SUVAT) equations

Solutions to Starter and E.g.s

No Exercise
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Summary
The equations for constant acceleration (SUVAT) are:

$v = u + at$	No s
$s = ut + \frac{1}{2}at^2$	No v
$v^2 = u^2 + 2as$	No t
$s = \frac{1}{2}(u + v)t$	No a
$s = vt - \frac{1}{2}at^2$	No u