Describing Vectors

Starter

1. (Review of GCSE material) State the position of the point $(2, -3)$ after being translated under the following vectors:

   (a) $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ (b) $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$

2. (Review of GCSE material) Find the magnitude of the vector $\begin{pmatrix} 7 \\ -24 \end{pmatrix}$.

Notes

Component form of a vector, $\begin{pmatrix} a \\ b \end{pmatrix}$

- ***Top*** number is ***horizontal*** movement (positive to the right, negative to the left)
- ***Bottom*** number is ***vertical*** movement (positive up, negative down)

Notation: $\begin{pmatrix} a \\ b \end{pmatrix} \equiv a\mathbf{i} + b\mathbf{j}$ $\mathbf{i} \equiv x-$axis $\mathbf{j} \equiv y-$axis

Magnitude $\left| \begin{pmatrix} a \\ b \end{pmatrix} \right| = \sqrt{a^2 + b^2}$

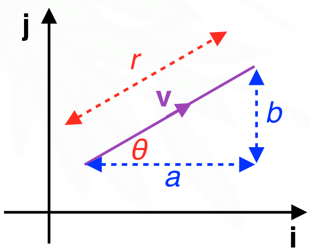
Direction of a vector

The direction of a vector is the angle from ***the positive $x-$axis measured anti-clockwise*** to the vector. This leads to the magnitude-direction form of a vector.

Magnitude-direction form of a vector

A vector can be defined by its magnitude and direction

Notation: (Magnitude, Direction) ***E.g.*** $(7, 90^o) \equiv 7\mathbf{j}$



$$(r, \theta) \equiv \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j}$$

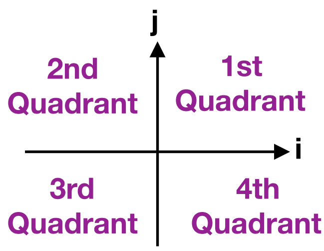
E.g. 1 Write the vector $(5, 60^o)$ in component form, using $\mathbf{i}$ and $\mathbf{j}$ notation.

Convert from component form to magnitude-direction form

To convert the vector $\begin{pmatrix} a \\ b \end{pmatrix} \equiv a\mathbf{i} + b\mathbf{j}$ to magnitude-direction form:

Magnitude is always $\left| \begin{pmatrix} a \\ b \end{pmatrix} \right| = \sqrt{a^2 + b^2}$

Direction: sketch a diagram of the vector to show where the angle is compared to the positive $x-$axis. It is important to know which quadrant the vector is in.



N.B. Ignore the signs of a and b when calculating the initial angle.

Values of a and b	Quadrant	Magnitude	Direction
$a > 0, b > 0$	1st	$\left\lvert\begin{pmatrix}a\\b\end{pmatrix}\right\rvert = \sqrt{a^2+b^2}$	$\tan^{-1}\left(\frac{b}{a}\right)$
$a < 0, b > 0$	2nd	$\left\lvert\begin{pmatrix}a\\b\end{pmatrix}\right\rvert = \sqrt{a^2+b^2}$	$180^o - \tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)$
$a < 0, b < 0$	3rd	$\left\lvert\begin{pmatrix}a\\b\end{pmatrix}\right\rvert = \sqrt{a^2+b^2}$	$180^o + \tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)$
$a > 0, b < 0$	4th	$\left\lvert\begin{pmatrix}a\\b\end{pmatrix}\right\rvert = \sqrt{a^2+b^2}$	$360^o - \tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)$

E.g. 2 Write $-5\mathbf{i}+4\mathbf{j}$ in magnitude-direction form.

Working: $|-5\mathbf{i}+4\mathbf{j}| = \sqrt{(-5)^2+4^2} = \sqrt{41}$

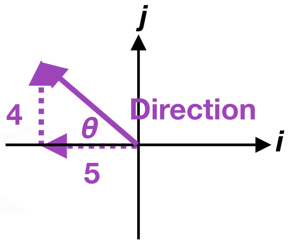
$-5\mathbf{i}+4\mathbf{j}$ is in the 2nd quadrant

$\theta = \tan^{-1}\frac{4}{5} = 38.66^o$

So direction is $180^o - 38.66^o = 141.3^o$

N.B. Direction measured anti-clockwise from positive $x-$axis

$-5\mathbf{i}+4\mathbf{j} \equiv \left(\sqrt{41},\ 141.3^o\right)$



Parallel vectors — are multiples of each other
For example, $\mathbf{v}$ is parallel to $6\mathbf{v}$, $\mathbf{i}+3\mathbf{j}$ is parallel to $4\mathbf{i}+12\mathbf{j}$, but $2\mathbf{i}-5\mathbf{j}$ and $4\mathbf{i}+10\mathbf{j}$ are not parallel to each other.

E.g. 3 Given that the vectors $-3\mathbf{i}+8\mathbf{j}$ and $6\mathbf{i}+x\mathbf{j}$ are parallel, find the value of x.

Unit vectors
A unit vector is a vector whose magnitude is 1.
Notation: the unit vector of $\mathbf{v}$ is $\hat{\mathbf{v}}$

$\mathbf{v} = |\mathbf{v}|\hat{\mathbf{v}}$ so $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|}$

E.g. 4 Let $\mathbf{v} = 3\mathbf{i}+4\mathbf{j}$. Find the unit vector in the direction of $\mathbf{v}$.

E.g. 5 Let $\mathbf{v} = 5\mathbf{i}-12\mathbf{j}$. Find:
(a) the unit vector and
(b) the vector of length 26 units in the direction of $\mathbf{v}$.

Working: (a) $\hat{\mathbf{v}} = \frac{5\mathbf{i}-12\mathbf{j}}{\sqrt{5^2+(-12)^2}} = \frac{1}{13}(5\mathbf{i}-12\mathbf{j})$

(b) Vector of length 26 units = $26 \times \frac{1}{13}(5\mathbf{i}-12\mathbf{j}) = 2(5\mathbf{i}-12\mathbf{j})$

Vector of specific magnitude a vector of magnitude k in the direction of $\mathbf{v}$ is $k\hat{\mathbf{v}} = k\dfrac{\mathbf{v}}{|\mathbf{v}|}$

Video: What is a vector and scalar quantity?
Video: Magnitude of a vector
Video: Magnitude and direction of a vector
Video: Unit vectors

Solutions to Starter and E.g.s

Exercise
p225 12A Qu 2i, 3i, 4-9

Summary

Component form of a vector: $\begin{pmatrix} a \\ b \end{pmatrix} \equiv a\mathbf{i} + b\mathbf{j}$ $\quad \mathbf{i} \equiv x-$axis $\quad \mathbf{j} \equiv y-$axis

Top number is ***horizontal*** movement (positive to the right, negative to the left) ***Bottom*** number is ***vertical*** movement (positive up, negative down)

Magnitude: $\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2 + b^2}$

Direction of a vector: the angle from ***the positive $x-$axis measured anti-clockwise*** to the vector.

Magnitude-direction form of a vector: $(r, \theta) \equiv \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j}$

Parallel vectors: multiples of each other

Unit vectors, $\hat{\mathbf{v}}$: have a magnitude of 1 $\quad \mathbf{v} = |\mathbf{v}|\hat{\mathbf{v}}$ so $\hat{\mathbf{v}} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

Vector of specific magnitude: a vector of magnitude k in the direction of $\mathbf{v}$ is $k\hat{\mathbf{v}} = k\dfrac{\mathbf{v}}{|\mathbf{v}|}$