

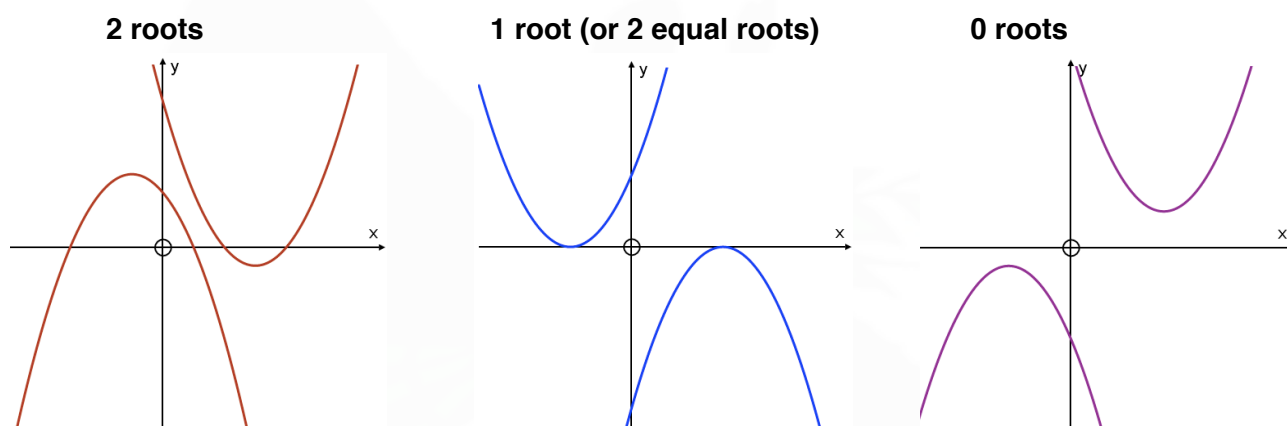
Discriminant

Starter

- (Review of last lesson)** Find the set of values that satisfies $3x + 5 < 17$ and $x^2 - 7x + 10 < 0$.
- (Review of last lesson)** Solve the inequality $6 + \frac{5}{x} \geq \frac{8}{x}$.
- How many roots can a quadratic equation have? Draw diagrams to illustrate your answer.

Notes

A quadratic curve can have 2, 1 or 0 roots. Each graph below has 2 examples of quadratic curves with the allotted number of roots.



The number of roots of a quadratic curve comes from the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The \pm part gives us the two roots.

- When $b^2 - 4ac > 0 \Rightarrow 2$ (real and distinct) roots
- When $b^2 - 4ac = 0 \Rightarrow 1$ (real) root or 2 equal (real roots)
- When $b^2 - 4ac < 0 \Rightarrow 0$ (real) roots

N.B. $b^2 - 4ac$ is so important it has a special name — discriminant and a special symbol, Δ (capital delta) so $\Delta = b^2 - 4ac$
The phrase “determine the nature of the roots” means “tell us how many roots there are”. It is a good idea to write down the values of a , b and c before substituting.

E.g. 1 Without solving the equation, determine the nature of the roots:

- (a) $x^2 - 6x + 4 = 0$ (b) $3x^2 + 4x + 2 = 0$

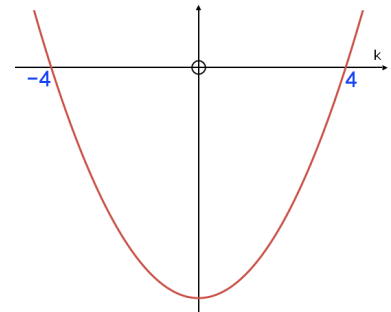
Working: (a) $a = 1$ $b = -6$ $c = 4$
 $b^2 - 4ac = (-6)^2 - 4 \times 1 \times 4 > 0$ *(exact value not needed)*
 Since discriminant > 0 , 2 real roots

E.g. 2 If the roots of $3x^2 + kx + 12 = 0$ are equal, find k .

N.B. Often discriminant questions will require you to solve a quadratic inequality. Remember to use set notation when giving the answer.

E.g. 3 Find the range of values of k for which $x^2 - kx + 4 = 0$ has two real and distinct roots.

Working: Two real and distinct roots $\Rightarrow b^2 - 4ac > 0$
 $a = 1, b = -k, c = 4$
 $(-k)^2 - 4 \times 1 \times 4 > 0$
 $\Rightarrow k^2 - 16 > 0$
Roots are $k = -4$ and $k = 4$



Coefficient of k^2 is +ve so concave-up
 $> 0 \Rightarrow$ above the x -axis

We need the k -values **to the left of -4** and **to the right of 4** .

$$\{k : k < -4\} \cup \{k : k > 4\}$$

E.g. 4 Find the range of values of k for which the curve $y = 2x^2 - (k + 1)x + 2$ has no real roots.

E.g. 5 The height of a ball, h m, at time t seconds is given by $h = 6 + 20t - 4.9t^2$. Prove that the ball does not reach a height of 32 m.

Video: [Discriminant](#)

[Solutions to Starter and E.g.s](#)

Exercise

p48 3E Qu 1i, 2i, 3i, 4-10

Summary

$\Delta = b^2 - 4ac$ is the discriminant of the quadratic.

When $b^2 - 4ac > 0 \Rightarrow 2$ (real and distinct) roots

When $b^2 - 4ac = 0 \Rightarrow 1$ (real) root or 2 equal (real roots)

When $b^2 - 4ac < 0 \Rightarrow 0$ (real) roots