Disproof by counter-example

Starter

1. ***(Review of last lesson)*** Consider the following statements:

A: The integer n is an even multiple of 5

B: The integer n has a final digit 0

C: The integer n has a pair of prime factors which differ by 3

(a) Use the symbols $\Leftrightarrow$ and $\Rightarrow$ to connect the statements $A, B,$ and C.

(b) Use the symbol $\not\Rightarrow$ to connect these statements:

2. Find a counter-example to prove that the statement "$\frac{1}{n^2} < \frac{1}{n}$ for $n \neq 0$" is not always true.

Notes

When trying to prove a statement is false, it is enough to find ***one example*** for which the ***statement does not hold true***. This is called ***disproof by counter-example***.

Often it can be a question of trial and error until a counter-example is found.

E.g. 1 Prove by counterexample that the statement "$2^x > 2x$ for $x \geq 0$" is not true.

Working: When $x = 1$, $2^1 = 2$ and $2 \times 1 = 2$ so $2^x = 2x$ so $2^x \not> 2x$.
Therefore the statement "$2^x > 2x$ for $x \geq 0$" is not true.

E.g. 2 Give a counter-example to show that "$(x + y)^2 > x + y$ for $x > 0$ and $y > 0$" is not always true.

E.g. 3 "The square of a number plus 1 is prime." Disprove this statement.

Video: Proof by counter example

Exam questions: Proof by counter example

Solutions to Starter and E.g.s

Exercise

p9 1C Qu 1-7, (8 red), 9

Summary

Disproof by counter-example: find ***one example*** for which the ***statement does not hold true***.