

Equation of a straight line

Starter

- (Review of last lesson)** The point $A(2, 5)$ lies on the line, L , with equation $y = 2x + 1$. Let B also lie on L .

 - Given that the x -coordinate of B is t , write down an expression for the y -coordinate of B in terms of t .
 - Hence find the coordinates of two points which lie on L and whose distance from A is $3\sqrt{5}$ units.
- (Review of GCSE material)** Find the equation of the line passing through $(-3, 5)$ and $(4, -9)$.
- (Review of GCSE material)** By using $y = mx + c$ and substituting to find c , derive the formula of the straight line which has gradient m and passes through the point (x_1, y_1) .

Notes

From GCSE:

Equation of a straight line of gradient, m , and y -intercept of c : $y = mx + c$

Gradient of line passing through (x_1, y_1) and (x_2, y_2) : $\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of line with gradient m and passing through (x_1, y_1) : $y - y_1 = m(x - x_1)$

N.B. The **gradient** of **horizontal** lines is **zero**.
The **gradient** of **vertical** lines is **undefined**.

E.g. 1 Find the equation of the line passing through the given points, expressing your answer in the form $ax + by = c$ where a , b and c are integers.

- $(10, -4)$ and $(-5, -12)$
- $(-4, 2)$ and $(-1, -3)$

Working: (a) Label the points: $(10, -4)$ and $(-5, -12)$

$$\text{Gradient, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\overset{(x_1, y_1)}{-12} - \overset{(x_2, y_2)}{-4}}{-5 - 10} = \frac{-8}{-15} = \frac{8}{15}$$

Substitute $y - y_1 = m(x - x_1)$: $y + 12 = \frac{8}{15}(x + 5)$

Multiply by 15 and expand: $15y + 180 = 8x + 40$
The equation of the line is $8x - 15y = 140$.

E.g. 2 Find the point of intersection between these pairs of straight lines:

- (a) $3x + 4y = 33$ $2y = x - 1$
(b) $2x + 3y = 7$ $6x = 11 - 9y$
(c) $y = mx + c$ $y = -mx + d$

Working: (a) $2y = x - 1 \Rightarrow x = 2y + 1$
Substitute into $3x + 4y = 33$: $3(2y + 1) + 4y = 33$
 $y = 3$
When $y = 3$: $x = 2 \times 3 + 1 = 7$
The point of intersection is $(7, 3)$.

E.g. 3 The line $L_1 : y = 2x - 5$ intersects the x -axis at the point P . The line $L_2 : x + y = 6$ intersects the x -axis at the point Q . Lines L_1 and L_2 intersect the at the point R . Calculate the area of the triangle PQR .

Video: [Equation of a straight line given the gradient and a point](#)

Video: [Equation of a straight line](#)

[Solutions to Starter and E.g.s](#)

Exercise

p95 6B Qu 1i, 2i, 3i, 4i, 5-9, (10 red)

Summary

Equation of a straight line of gradient, m , and y -intercept of c : $y = mx + c$

Gradient of line passing through (x_1, y_1) and (x_2, y_2) : $\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of line with gradient m and passing through (x_1, y_1) : $y - y_1 = m(x - x_1)$