Equations of Normals to Curves

Starter

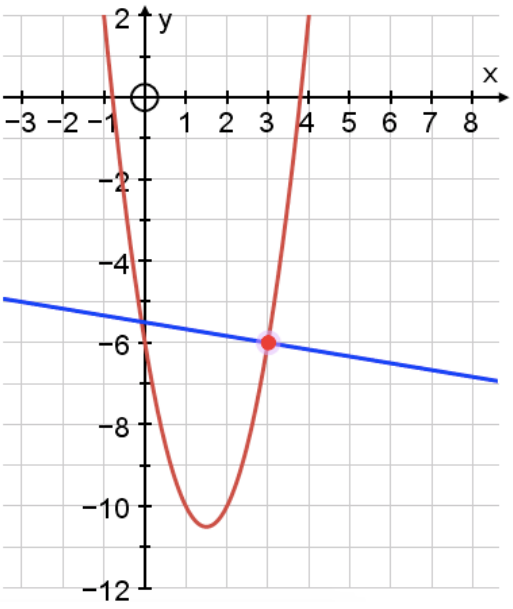
For ***perpendicular lines***, the product of the gradients is -1 i.e. $m_1 \times m_2 = -1$

1. **(Review of GCSE material)** Fill in the table with the perpendicular gradients.

Gradient	Perpendicular gradient
5	
-7	
$\frac{1}{8}$	
$-\frac{2}{3}$	
$-3\frac{4}{7}$	
4.6	

N.B. When a line is ***normal to a curve***, it means that is is ***perpendicular*** to it at the the point where they intersect.

2. Find the equation of the ***normal*** to the curve $y = 2x^2 - 6x - 6$ at the point $(3, -6)$. Express your answer in the form $ax + by + c = 0$, where a, b and c are integers.



Notes

Perpendicular gradients

The gradient of the normal is the ***negative reciprocal*** of the gradient of the tangent.

If $\frac{a}{b}$ is the gradient of the tangent, the gradient of the normal is $-\frac{b}{a}$

"Flip it" and change the sign.

Success criteria — finding the equation of the normal

1. Differentiate the function.
2. Find the gradient at the point given.
3. Find the negative reciprocal of the gradient — this will be the m in the formula.
4. If necessary, find the $y-$value by substituting back into the original equation.
5. Use either $y = mx + c$ or the formula $y - y_1 = m(x - x_1)$.

N.B. When the gradient of the tangent is 0 $\Rightarrow$ tangent parallel to the $x-$axis. So the normal to the curve is parallel to the $y-$axis (i.e. it is vertical and has the form $x =$ "a number").

E.g. 1 Find the equation of the normal to the curve $y = x^3 - 4x$ when $x = 3$. Express your answer in the form $ax + by = c$ where a, b and c are integers.

Working:

$\frac{dy}{dx} = 3x^2 - 4$

When $x = 3$, $\frac{dy}{dx} = 3 \times 3^2 - 4 = 23$ ***gradient of tangent***

So gradient of normal is $-\frac{1}{23}$ ***negative reciprocal***

When $x = 3$, $y = 3^3 - 4 \times 3 = 15$

So gradient is $-\frac{1}{23}$, passes through $(3, 15)$

Using $y - y_1 = m(x - x_1)$: $y - 15 = -\frac{1}{23}(x - 3)$

Multiply by 23 and rearrange: $x + 23y = 348$

E.g. 2 Find the equation of the normal to the curve $y = x^2 - 4x - 3$ at the point where the curve cuts the $y-$axis. Express your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Video: Equation of tangents/normals

Solutions to Starter and E.g.s

Exercise

Normal: p274 14A Qu 1i, 3, 4, 8, 10, 13 (not 12)

Summary

Perpendicular gradients: ***negative reciprocal***

If $\frac{a}{b}$ is the gradient of the tangent, the gradient of the normal is $-\frac{b}{a}$

Finding the equation of the normal

1. Differentiate the function.
2. Find the gradient at the point given.
3. Find the negative reciprocal of the gradient — this will be the m in the formula.
4. If necessary, find the $y-$value by substituting back into the original equation.
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N.B. When the gradient of the tangent is 0 $\Rightarrow$ tangent parallel to the $x-$axis. So the normal to the curve is parallel to the $y-$axis (i.e. it is vertical and has the form $x =$ "a number").