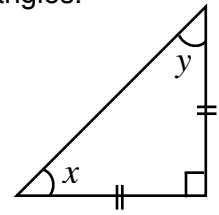


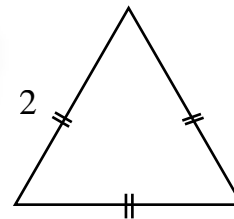
Exact values of trigonometric functions

Starter

1. **(Review of last lesson)** Consider the right-angled isosceles triangle with angles.
- State the size of angles x and y .
 - Given that the length of the opposite and adjacent sides are 1 unit, calculate the length of the hypotenuse.
 - Hence write down the values of $\tan 45^\circ$, $\sin 45^\circ$ and $\cos 45^\circ$. Rationalise the denominator where necessary.



2. Draw an equilateral triangle of side 2 units.
- Calculate the height of the triangle.
 - Hence write down the values of:
 - $\tan 60^\circ$, $\sin 60^\circ$ and $\cos 60^\circ$
 - $\tan 30^\circ$, $\sin 30^\circ$ and $\cos 30^\circ$
- Rationalise the denominator where necessary.



Notes

E.g. 1 Collate the values from the starter in the table below and using your knowledge of the trigonometric graphs, complete the rest of the table:

	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin									
cos									
tan									

N.B. These values are not given on the formula page so you will need to learn them.

To get the tan values do $\frac{\sin}{\cos}$:

$$\text{E.g. } \tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

E.g. 2 Using your knowledge of trigonometric graphs, and without using a calculator, find the exact value of:

- (a) $\sin 390^\circ$ (b) $\cos 240^\circ$ (c) $\tan 315^\circ$

Working: (a) $\sin 390^\circ = \sin(390^\circ - 360^\circ) = \sin 30^\circ = \frac{1}{2}$

E.g. 3 Without using a calculator, find the value of:

- (a) $\cos^2 45^\circ + \sin^2 45^\circ$ (b) $2 \sin 150^\circ \cos 150^\circ$

Working: (a) $\cos^2 45^\circ + \sin^2 45^\circ = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = 1$

E.g. 4 Find the value of θ , where $0^\circ \leq \theta \leq 90^\circ$, such that:

- (a) $\sin \theta \cos 315^\circ + \sin 45^\circ \cos 30^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$
 (b) $\cos^2 \theta + \sin^2 240^\circ = 1$
 (c) $\tan \theta = \frac{2 \tan 45^\circ}{1 - \tan^2 45^\circ}$

Working: (a) $\cos 315^\circ = \cos(315^\circ - 360^\circ) = \cos(-45^\circ) = \cos 45^\circ = \frac{\sqrt{2}}{2}$

$$\sin \theta \times \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

Multiply by 4: $\sin \theta \times 2\sqrt{2} + \sqrt{6} = \sqrt{6} + \sqrt{2}$

$$\sin \theta = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$$

$$\therefore \theta = 30^\circ$$

Video:

[Trigonometric ratios for 30, 45 and 60 degrees](#)

Video:

[Trigonometric ratios for 30, 45 and 60 degrees](#)

[Solutions to Starter and E.g.s](#)

Exercise

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Summary

These are the exact values you need to learn:

	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0