Finding the Gradient at a Point

Starter

1. **(Review of last lesson)**

   Differentiate: (a) $f(x) = -\dfrac{5}{2\sqrt[8]{x^3}}$ (b) $y = \dfrac{15+7x}{8x^3}$

2. Find the gradient of the curve $y = 5x^2 + 4x - 8$ where $x = 2$.

3. Write down a method for finding the gradient of a curve at a particular point.

4. Find the coordinates of the point(s) on $y = (2x-5)(x+1)$ where the gradient is -3.

5. Write down a method for finding the coordinates point(s) at which the gradient of a curve has a specific value.

Notes

From the starter:

Success Criteria — finding the gradient at a particular point

1. Differentiate to find the derivative.
2. Substitute the $x-$value into the derivative to find the value of the gradient.

Success Criteria — finding a point with a specific gradient

1. Differentiate to find the derivative.
2. Put the derivative equal to the value of the gradient.
3. Solve the equation to find the $x-$coordinate.
4. Substitute the $x-$coordinate into the equation for y (original equation).

E.g. 1 Find the gradient of the curve at the given point:

(a) $s = \sqrt{t}(1+\sqrt{t})$ when $t = 4$ (b) $y = \dfrac{x^2-4}{x}$ when $x = -2$

Working: (a) $s = \sqrt{t}(1+\sqrt{t}) = \sqrt{t} + t = t^{\frac{1}{2}} + t$

$$\frac{ds}{dt} = \frac{t^{-\frac{1}{2}}}{2} + 1 = \frac{1}{2t^{\frac{1}{2}}} + 1 = \frac{1}{2\sqrt{t}} + 1$$

When $t = 4$, $\dfrac{ds}{dt} = \dfrac{1}{2\sqrt{4}} + 1 = \dfrac{1}{4} + 1 = \dfrac{5}{4}$

E.g. 2 Find the coordinates of the point(s) on the given curve where the gradient has the value specified:

(a) $s = t^3 - 12t + 9$; gradient = 15 (b) $y = 3 - \frac{2}{x}$; gradient = $\frac{1}{2}$

Working: (a) $\frac{ds}{dt} = 3t^2 - 12$

$\frac{ds}{dt} = 15$ when $3t^2 - 12 = 15 \quad \Rightarrow \quad 3t^2 = 27$

$t^2 = 9$ so $\quad \therefore t = \pm 3$

When $t = 3$, $s = 3^3 - 12 \times 3 + 9 = 0$

When $t = -3$, $s = (-3)^3 - 12 \times (-3) + 9 = 18$

The curve has gradient 15 at $(3, 0)$ and $(-3, 18)$

Video: Gradient of a curve at a point

Solutions to Starter and E.g.s

Exercise

p264 13E Qu 3i, 4i, 6i, 11, 14

Summary

Finding the gradient at a particular point

1. Differentiate to find the derivative.
2. Substitute the $x-$value into the derivative to find the value of the gradient.

Finding a point with a specific gradient

1. Differentiate to find the derivative.
2. Put the derivative equal to the value of the gradient.
3. Solve the equation to find the $x-$coordinate.
4. Substitute the $x-$coordinate into the equation for y (original equation).