

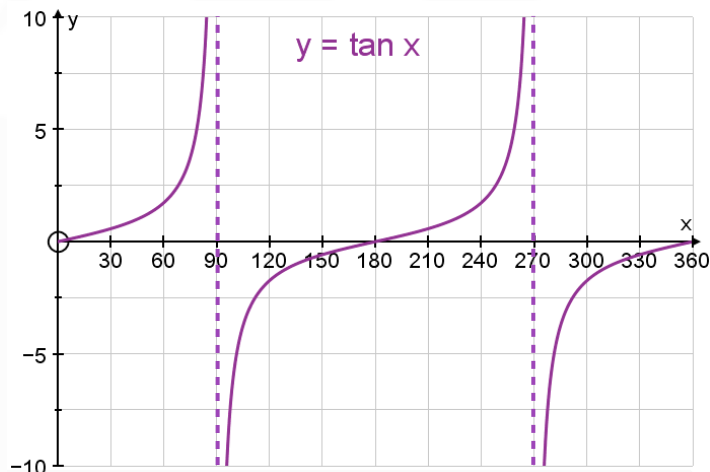
Graph of tangent

Starter

1. **(Review of last lesson)** Two sides of a triangle have lengths $(2x + 1)$ and $(x + 4)$. Given that the angle between the sides is 60° and that the area of the triangle is $20\sqrt{3}$, find the value of x to 3 s.f..

Notes

The tangent graph was met at GCSE level. The dotted lines are **asymptotes** i.e. the graph gets closer and closer to the asymptote but never actually touches it. So $\tan x$ is not defined when $x = 90^\circ$ and $x = 270^\circ$.



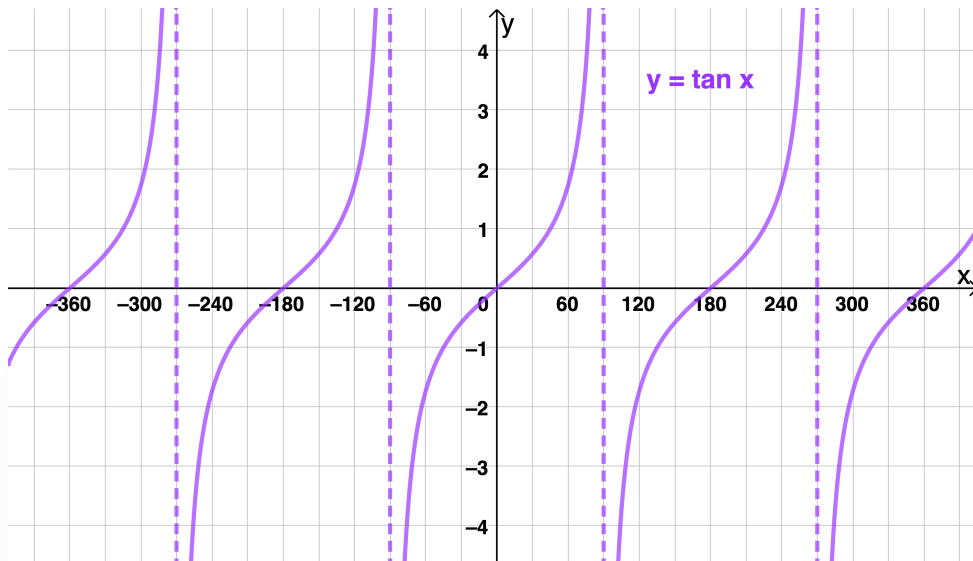
- E.g. 1** (a) Using the tangent graph, find an angle θ , where $0^\circ \leq \theta \leq 360^\circ$, such that:
(i) $\tan 60^\circ = \tan \theta$ (ii) $\tan 120^\circ = \tan \theta$ (iii) $\tan 320^\circ = \tan \theta$
- (b) Hence, complete $\tan x \equiv \tan(\dots)$ with an expression involving x in the bracket.

Working: (a) (i) $\tan 60^\circ = \tan 240^\circ$

Extending beyond $0^\circ \leq x \leq 360^\circ$

Similar to the sine and cosine curves, the tangent can be extended for angles below 0° and above 360° . Asymptotes are at $x = \pm 90^\circ, \pm 270^\circ, \pm 450^\circ, \pm 630^\circ$ etc.

While sine and cosine have a period of 360° , the tangent graph repeats itself every 180° .



E.g. 2 Using the extended tangent graph of find three values for θ angle such that:

- (a) $\tan 45^\circ = \tan \theta$ (b) $\tan 150^\circ = \tan \theta$ (c) $\tan(-160^\circ) = \tan \theta$

Working: (a) $\tan 45^\circ = \tan 225^\circ = \tan 405^\circ = \tan(-135^\circ) = \tan(-315^\circ)$
Only three angles are required.

- E.g. 3** (a) Given that $\tan 40^\circ = 0.839$, without using a calculator, state the value of:
(i) $\tan 220^\circ$ (b) $\tan(-40^\circ)$ (c) $\tan 140^\circ$
(b) Hence state the relationship between $\tan(-\theta)$ and $\tan \theta$.

Working: (a) (i) $\tan 220^\circ = \tan(220^\circ - 180^\circ) = \tan 40^\circ = 0.839$

E.g. 4 In a right-angled triangle, the tangent of an angle is given by $\frac{\text{opp}}{\text{adj}}$. State the fractions for

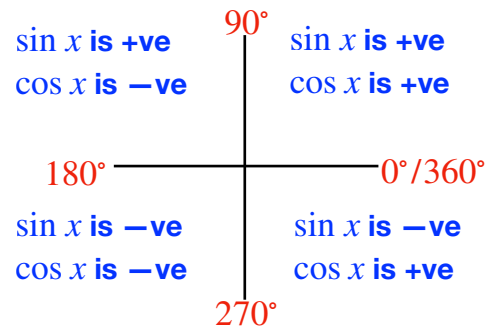
sine and cosine and by doing $\frac{\sin \theta}{\cos \theta}$, find the relationship between sine, cosine and tangent.

E.g. 5 By considering the graph of $y = \tan x$, decide whether the curve is positive or negative for the range of values in the table.

Ratio	$0^\circ < x < 90^\circ$	$90^\circ < x < 180^\circ$	$180^\circ < x < 270^\circ$	$270^\circ < x < 360^\circ$
$\tan x$				

E.g. 6 Using your table from **E.g. 1**, write in each quadrant whether $\tan x$ is positive or negative for the range of values within that quadrant.

N.B. Negative angles are measured clockwise.



Video: [Trigonometric graphs](#)

[Solutions to Starter and E.g.s](#)

Exercise

p176 10B Qu 1b, 2i, 3, 4, 5-6, (7 red)

Summary

$$\tan x \equiv \tan(x \pm 180^\circ)$$

$$\tan(-\theta) = -\tan \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

When sine, cosine and tangent are positive and negative:

