Graphs of sine and cosine

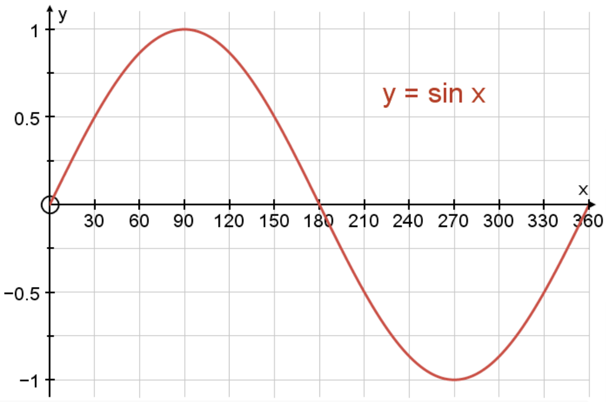
Starter

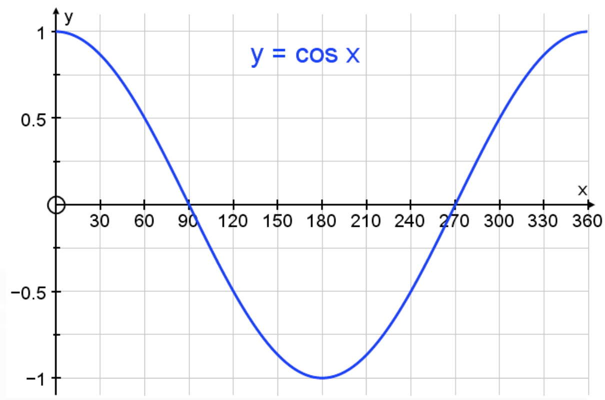
1. *(Review of last lesson)* If x is small enough that x^2 and higher powers of x can be neglected, show that the function $(x-2)(1+3x)^8$ has a linear approximation of the form $a+bx$, where a and b are to be found.

2. Simplify $(1-x)^8+(1+x)^8$ and hence find the exact value of $0.99^8+1.01^8$

Notes

You will be familiar with the shape of the sine and cosine graphs from your GCSE course.





E.g. 1 (a) Using the symmetry of the sine graph, find a second angle that gives the same value as:

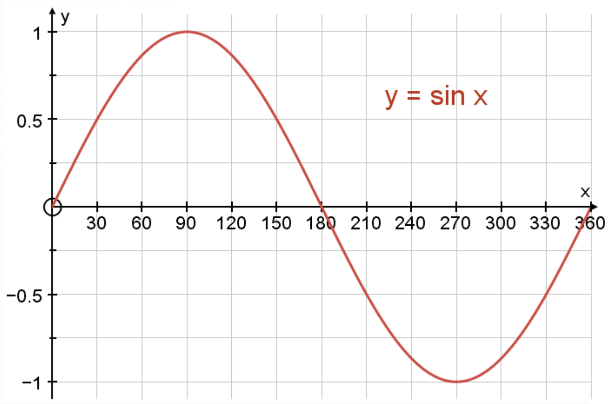
(i) $\sin 30°$

(ii) $\sin 60°$

(iii) $\sin 135°$

(b) Hence, copy and complete with an expression involving x in the bracket:

$$\sin x \equiv \sin(\dots).$$



Working: (a) (i) $\sin 30° \equiv \sin 150°$

E.g. 2 (a) Using the symmetry of the cosine graph, find a second angle that gives the same value as:

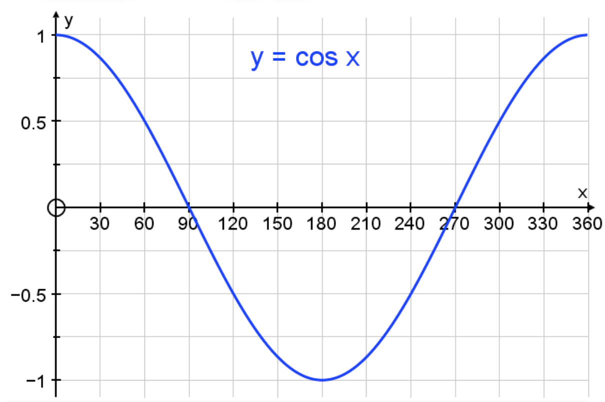
(i) $\cos 60°$

(ii) $\cos 150°$

(iii) $\cos 240°$

(b) Hence, copy and complete with an expression involving x in the bracket:

$$\cos(180-x) \equiv \cos(\dots).$$



Working: (a) (i) $\cos 60° = \cos 300°$

E.g. 3 (a) By comparing the graphs of sine and cosine, find an angle θ, where $0^\circ \leq \theta \leq 90^\circ$, such that:

(i) $\sin 30^\circ = \cos\theta$ (ii) $\cos 80^\circ = \sin\theta$

(iii) $\sin 55^\circ = \cos\theta$ (iv) $\cos 25^\circ = \sin\theta$

(b) Hence, copy and complete the following with an expression involving x in the bracket $\sin x \equiv \cos(\ldots)$ and $\cos x \equiv \sin(\ldots)$.

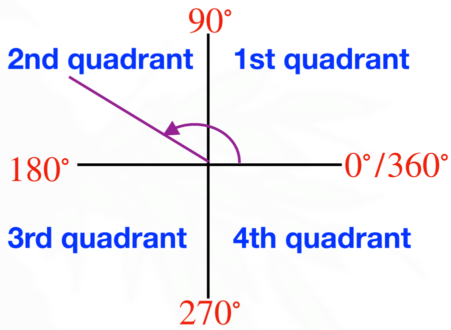
Working: (a) (i) $\sin 30^\circ = \cos 60^\circ$

The ***amplitude*** is the ***distance from the centre line***, i.e. the $x-$axis, ***to the maximum/minimum***.
So the amplitude of both $y = \sin x$ and $y = \cos x$ is 1.
In addition: $-1 \leq \sin x \leq 1$ $-1 \leq \cos x \leq 1$

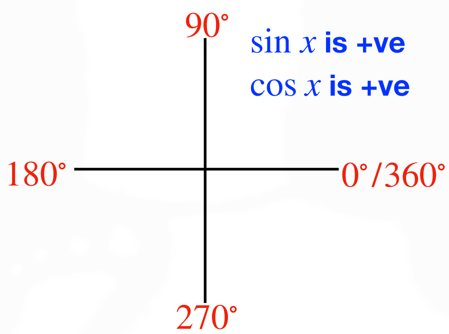
E.g. 4 By considering the graphs of $y = \sin x$ and $y = \cos x$ decide whether each curve is positive or negative for the range of values in the table.

Ratio	$0^\circ < x < 90^\circ$	$90^\circ < x < 180^\circ$	$180^\circ < x < 270^\circ$	$270^\circ < x < 360^\circ$
$\sin x$				
$\cos x$				

E.g. 5 Consider a set of axes which create four quadrants. Let angles be measured ***anti-clockwise from the positive*** $x-$***axis***.
Using your table from ***E.g. 1***, write in each quadrant whether $\sin x$ and $\cos x$ are positive or negative for the range of values within that quadrant.
N.B. Negative angles are measured clockwise.



Working:



This diagram will become useful when solving trigonometric equations in future lessons.

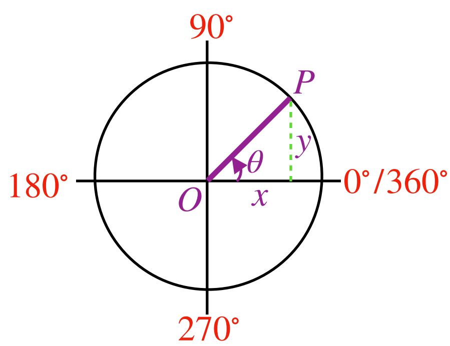
The Unit Circle

E.g. 6 The unit circle has a radius of 1 unit and is centred on the origin.

(a) State the equation of the unit circle in Cartesian form.

(b) The point P lies on the circumference in the first quadrant of the unit circle. The line OP, where O is the origin, makes an angle θ with the positive $x-$axis. State the $x-$ and $y-$coordinates of the point P.

(c) By substituting your answers for (b) into the



equation of the circle from (a), derive a
trigonometric identity involving sine and cosine.

Hence, as we go round the unit circle, the $x-$coordinate gives the values for the cosine graph and the $y-$coordinate gives the values for the sine graph.

Interactive unit circle - sine & cosine
Interactive unit circle - all

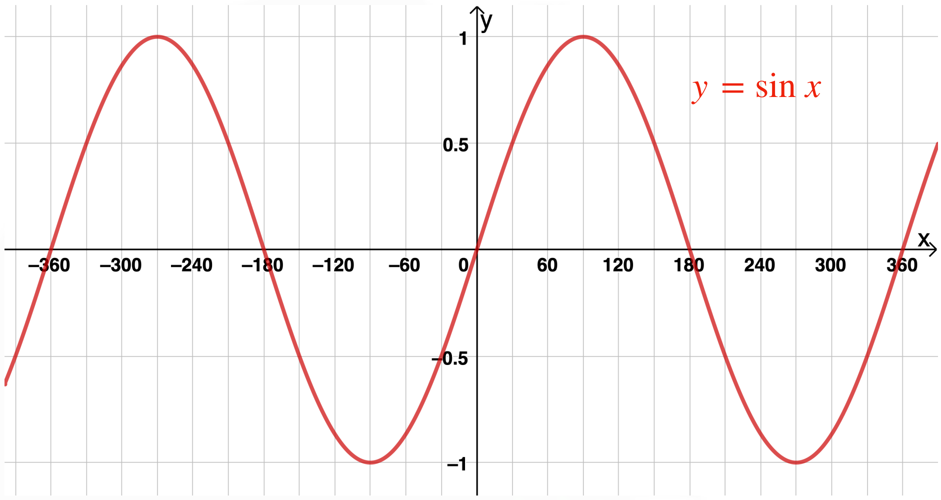
Extending beyond $0^\circ \leq x \leq 360^\circ$

While the majority of angles considered are $0^\circ \leq x \leq 360^\circ$, there is no reason why the graph cannot be extended both below 0° and above 360°.

Periodic nature of sine and cosine graphs

Let the point Q be on the circumference of the unit circle such that OQ makes an angle of 30° with the positive $x-$axis. The periodic nature of the sine graph means that if the angle were 390°, the point Q would be in the same position. This leads to the results:

$\sin x \equiv \sin(x \pm 360^\circ)$ and $\cos x \equiv \cos(x \pm 360)$

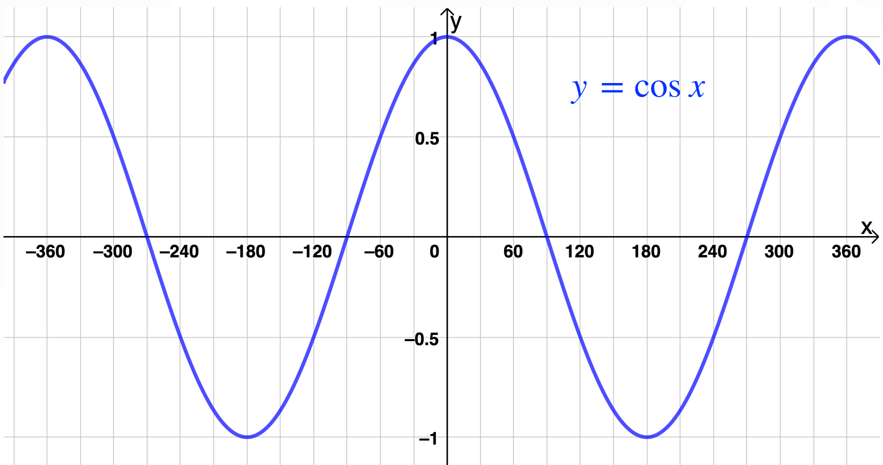


E.g. 7 (a) By considering the 'extended' sine graph, find the value of θ, where $0^\circ \leq \theta \leq 90^\circ$, such that:

(i) $\sin(-30^\circ) \equiv -\sin\theta$ (ii) $\sin(-60^\circ) \equiv -\sin\theta$

(b) Copy and complete the statement with an expression in terms of x in the bracket: $\sin(-x) \equiv -\sin(\ldots)$.

Working: (a) (i) $\sin(-30^\circ) \equiv -\sin 30^\circ$



E.g. 8 (a) By considering the 'extended' sine graph, find the value of θ, where $0^\circ \leq \theta \leq 180^\circ$, such that:

(i) $\cos(-60^\circ) \equiv \cos\theta$ (ii) $\cos(-135^\circ) \equiv \cos\theta$

(b) Copy and complete the statement with an expression in terms of x in the bracket: $\cos(-x) \equiv \cos(\ldots)$.

Working: (a) (i) $\cos(-60^\circ) \equiv \cos 60^\circ$

Video: Trigonometric graphs

Solutions to Starter and E.g.s

Exercise
p175 10A Qu 1i, 2i, 3i, 4i, 5i, 6-9, (10 red)

Summary
Identities involving $360°$: $\sin x \equiv \sin(x \pm 360°)$ and $\cos x \equiv \cos(x \pm 360)$
Identities involving $180°$: $\sin x \equiv \sin(180° - x)$ and $\cos(180 + x) \equiv \cos(180 - x)$
Identities involving $90°$: $\sin x \equiv \cos(90 - x)$ and $\cos x \equiv \sin(90 - x)$
Identities involving negative angles: $\sin(-x) \equiv -\sin x$ and $\cos(-x) \equiv \cos x$

When sine and cosine are positive and negative: