Interpreting Derivatives and Second Derivatives

Starter

1. **(Review of last lesson)** Find the exact value of the gradient of the tangent at the point where $x = 2$ on the curve $y = (2 - \sqrt{x})^2$.

N.B. The second derivative is found by differentiating twice and is denoted by $\frac{d^2y}{dx^2}$ or $f''(x)$.

2. Find the second derivative of the following functions:
   (a) $y = x^3$ (b) $y = x(4x^2 - x)$ (c) $y = 5x - 2$

Notes

1st derivative

Explanation: gradient of the function ≡ rate at which the function is changing

Notation: $\frac{dy}{dx}$ or $f'(x)$

- $\frac{dy}{dx}$ is the rate of change of y with respect to x.
- $\frac{ds}{dt}$ is the rate of change of s with respect to t.

2nd derivative

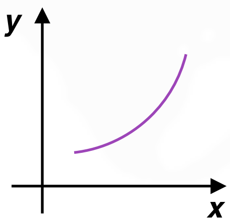
Explanation: gradient of the gradient ≡ rate at which the gradient is changing

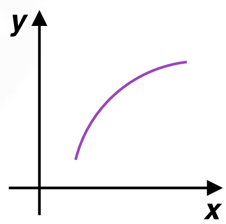
Notation: $\frac{d^2y}{dx^2}$ or $f''(x)$

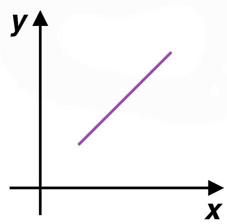
The second derivative can be thought of as "***the gradient of the gradient***".
Is the gradient increasing or decreasing?

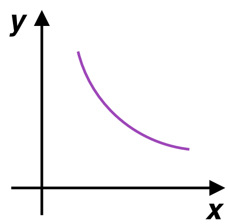
- If the ***gradient*** is ***increasing***, **second derivative** is ***positive***.
  E.g. The gradient could be increasing from 2 to 3 or from −6 to −5
- If the ***gradient*** is ***decreasing***, ***second derivative*** is ***negative***.
  E.g. The gradient could be decreasing from 8 to 7 or from −3 to −4

E.g. 1 Write down the signs of $f'(x)$ and $f''(x)$ for the following graphs of $y = f(x)$.

(a)


(b)


(c)


(d)


Working: (a) Curve is going up so $f'(x) > 0$
The gradient of the curve is increasing so $f''(x) > 0$

E.g. 2 Find the value of the 2nd derivative at the given value of x:

(a) $f(x) = x^3 - x^2, \ x = 3$ (b) $y = x\sqrt{x} - \dfrac{1}{x}, \ x = 4$

Working: (a) $f'(x) = 3x^2 - 2x \quad \Rightarrow \quad f''(x) = 6x - 2$
$f''(3) = 6 \times 3 - 2 = 16$

Video: Second derivative

Solutions to Starter and E.g.s

Exercise
p264 13E Qu 1, 2i, 5i

Summary

1st derivative: gradient of the function ≡ rate at which the function is changing $\dfrac{dy}{dx}$ or $f'(x)$

2nd derivative:gradient of the gradient ≡ rate at which the gradient is changing $\dfrac{d^2y}{dx^2}$ or $f''(x)$

If the ***gradient*** is ***increasing***, **second derivative** is ***positive***.
If the ***gradient*** is ***decreasing***, ***second derivative*** is ***negative***.