Kinematics and Calculus

Starter

1. **(Review of AS material)**
Find the gradient of the curve $x = t + 3t^2$ at the point where $t = -2$.

2. **(Review of AS material)**
Given that $\frac{dv}{dt} = t^3 + 2t - 1$ and that $v = 13$ when $t = 2$, find v as a function of t.

Notes

When ***acceleration*** is ***not constant***, it can be given as a function of time i.e. $a(t)$

N.B. When acceleration is not constant, the letter x is used for ***displacement***.

When acceleration is variable, we use differentiation and integration to move between displacement, velocity and acceleration.

	Displacement		**Velocity**		**Acceleration**
Differentiation:	x	***Differentiate*** →	$v = \frac{dx}{dt}$	***Differentiate*** →	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
Integration:	$x = \int v\,dt$	← ***Integrate***	$v = \int a\,dt$	← ***Integrate***	a

Displacement, velocity or acceleration could be given as the starting equation.

N.B. After integrating you will need to use the boundary condition to find the constant of integration, c.

E.g. 1 An object's displacement in metres at t second is given by $x = 2t^3 - 4t^2 + 3$.
(a) Calculate the object's displacement at time $t = 3$.
(b) Find the object's velocity equation.
(c) What is its velocity at time $t = 3$?
(d) Find its initial acceleration.

Working:

(a) When $t = 3$: $x = 2 \times 3^3 - 4 \times 3^2 + 3 = 21$ m

(b) $v = \frac{dx}{dt} = 6t^2 - 8t$

(c) When $t = 3$: $v = \frac{dx}{dt} = 6 \times 3^2 - 8 \times 3 = 30$ m/s

(d) $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12t - 8$
When $t = 0$: $a = -8$ m/s².

E.g. 2 During braking the speed of a car is modelled by $v = 40 - 2t^2$ (in m/s) until it stops moving.

(a) How long is it before the car stops moving?

(b) How far does it move before it stops? Give your answer to the nearest metre.

Video: Linear motion with variable acceleration

Linear motion with variable acceleration EQ

Solutions to Starter and E.g.s

Exercise

p431 19C Qu 1i, 3i, 5i, 6-9

Summary

When ***acceleration*** is ***not constant***, it can be given as a function of time i.e. $a(t)$

	Displacement		Velocity		Acceleration
Differentiation:	x	***Differentiate*** →	$v = \frac{dx}{dt}$	***Differentiate*** →	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
Integration:	$x = \int v\,dt$	← ***Integrate***	$v = \int a\,dt$	← ***Integrate***	a

Displacement, velocity or acceleration could be given as the starting equation.

N.B. After integrating you will need to use the boundary condition to find the constant of integration, c.