Laws of Logs

Starter

1. **(Review of last lesson)** State the value of: (a) $\log_9 \sqrt{3}$ (b) $\ln e$

2. **(Review of last lesson)** Rewrite $\log 100 = 2$ in index form.

3. **(Review of GCSE material)** Write down the 3 laws of indices.

Notes

How do the laws of indices translate into laws of logarithms?

Let $x = a^p$ and $y = a^q$.

Transferring to log from we have: $\log_a x = p$ and $\log_a y = q$.

Consider the 1st law of indices

$a^{p+q} = a^p \times a^q$ so $a^{p+q} = xy$ since $x = a^p$ and $y = a^q$

Transferring $a^{p+q} = xy$ from index form to logarithmic form: $p + q = \log_a xy$

But $p = \log_a x$ and $q = \log_a y$ so $\log_a x + \log_a y = \log_a xy$ (1st law of logs)

Consider the 2nd law of indices

$a^{p-q} = \dfrac{a^p}{a^q}$ $a^{p-q} = \dfrac{x}{y}$ since $x = a^p$ and $y = a^q$

Transferring $a^{p-q} = \dfrac{x}{y}$ from index form to logarithmic form: $p - q = \log_a \dfrac{x}{y}$

But $p = \log_a x$ and $q = \log_a y$ so $\log_a x - \log_a y = \log_a \dfrac{x}{y}$ (2nd law of logs)

Consider the 3rd law of indices

$(a^p)^n = a^{pn}$ so $x^n = a^{pn}$ since $x = a^p$

Transferring $x^n = a^{pn}$ from index form to logarithmic form: $\log_a x^n = np$

But $p = \log_a x$ so $\log_a x^n = n \log_a x$ (2nd law of logs)

Summary — 3 laws of logs

- $\log_a x + \log_a y = \log_a xy$ when ***adding*** logs, we ***multiply*** the values
- $\log_a x - \log_a y = \log_a \dfrac{x}{y}$ when ***subtracting*** logs, we ***divide*** the values
- $\log_a x^n = n \log_a x$ when ***raised to a power***, bring ***power to*** the ***front***

E.g. 1 Write these as a single logarithm:
(a) $\log_a 2 + \log_a 5$ (b) $2 \ln 7$ (c) $3 \log_a 2 - \log_a 4$

Working: (a) $\log_a 2 + \log_a 5 = \log_a (2 \times 5) = \log_a 10$ ***(1st law)***

E.g. 2 Write the following in terms of p, q and r where $p = \log_a x$, $q = \log_a y$ and $r = \log_a z$:

(a) $\log_a(x^2yz^3)$ (b) $\log_a\left(\dfrac{x}{y^3z}\right)$

Working: (a)
$$\begin{aligned}\log_a(x^2yz^3) &= \log_a x^2 + \log_a y + \log_a z^3 && \textit{(1st law)}\\ &= 2\log_a x + \log_a y + 3\log_a z && \textit{(3rd law)}\\ &= 2p + q + 3r\end{aligned}$$

E.g. 3 Solve these equations:

(a) $\log_{10} 4 + 2\log_{10} x = 2$ (b) $\log_3(x+11) - \log_3(x-5) = 2$

Working: (a)
$$\begin{aligned}\log_{10} 4 + \log_{10} x^2 &= 2 && \textit{(3rd law)}\\ \log_{10} 4x^2 &= 2 && \textit{(1st law)}\\ 4x^2 &= 10^2 && \textit{(log to index form)}\\ x^2 &= 25\\ x &= 5 \text{ (not } -5 \text{ since } \log(-5) \text{ does not exist)}\end{aligned}$$

E.g. 4 If $\log_x 2 = a$, $\log_x 3 = b$ and $\log_x 5 = c$, express these in terms of a, b and c.

(a) $\log_x 6$ (b) $\log_x 16$ (c) $\log_x 60$ (d) $\log_x \dfrac{15}{2}$

Working: (a)
$$\begin{aligned}\log_x 6 &= \log_x(2\times 3)\\ &= \log_x 2 + \log_x 3 && \textit{(1st law)}\\ &= a + b\end{aligned}$$

Video: Laws of logs

Solutions to Starter and E.g.s

Exercise

p121 7B Qu 1i, 2i, 3i, 4-12

Summary

Laws of logs:

- $\log_a x + \log_a y = \log_a xy$ — when ***adding*** logs, we ***multiply*** the values
- $\log_a x - \log_a y = \log_a \dfrac{x}{y}$ — when ***subtracting*** logs, we ***divide*** the values
- $\log_a x^n = n\log_a x$ — when ***raised to a power***, bring ***power to*** the ***front***