Logarithms

Starter

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1. (Review of last lesson) Solve $3^{2x+1} - 12 \times 3^{x+2} + 729 = 0$.
   Hint: prepare the equation so that we can "Let $u = 3^x$"

2. Solve these equations: (a) $2^{x+4} = \dfrac{16^{3x}}{8^{1-5x}}$ (b) $2^x = 9$.
   N.B. For (a), express 16 and 8 as powers of 2, then use the laws of indices.
   If you can't do (b), don't spend *too* much time on it.

Notes

$\log_a c$ means "**to what power must a be raised to get c**"

This phrase is ***important***:

$$\log_a c \equiv \text{ to what power must } a \text{ be raised to get } c$$

a is called the ***base*** of the logarithm

E.g. 1 By using the phrase above, state the values of:
(a) $\log_a a$ (b) $\log_a 1$ (c) $\log_2 8$ (d) $\log_3 9$ (e) $\log_{25} 5$.
Give reasons for your answers using indices.

Working: (a) $\log_a a$ $\Rightarrow$ To what power must a be raised to get a
$\log_a a = 1$ since $a^1 = a$

So

$\log_2 8 = 3 \Leftrightarrow 2^3 = 8$
$\log_3 9 = 2 \Leftrightarrow 3^2 = 9$
$\log_{25} 5 = \dfrac{1}{2} \Leftrightarrow 25^{\frac{1}{2}} = 5$

Now re-write $\log_a c = b$ in index form.

$a^b = c$

$a^b = c \quad \Leftrightarrow \quad \log_a c = b$
"a b c" "log a c b"

$\text{Base}^{\text{Power}} = \text{Number} \qquad \log_{\text{Base}} \text{Number} = \text{Power}$

N.B. the base of the logarithm is also the number being raised to the power.

Logarithms and natural logarithms

Logarithms were invented by the Scottish mathematician John Napier, with the help of Henry Briggs about 1614. Also discovered independently by Joost Bürgi, a Swiss, at about the same time.

Logarithms to the base e, where $e \approx 2.718281828\ldots$, are called ***natural logarithms***. The number e is an irrational number.

Notation: $\ln x \equiv \log_e x$ — always use the $\ln$ notation
$\log x = \log_{10} x$ i.e. ***if no base is written, assume it is base 10***

N.B. In practice, the base of a logarithm is a positive integer greater than 1
Usually bases are e or 10

E.g. 2 Without using a calculator, state the values of:

(a) $\log_2 16$ (b) $\log 10000$ (c) $\log_5 \frac{1}{5}$ (d) $\log_7 \sqrt{7}$

Remember: ***if no base is written, assume it is base 10***

Working: (a) $\log_2 16$ $\Rightarrow$ To what power must 2 be raised to get 16
$\log_2 16 = 4$

E.g. 3 Rewrite in logarithm form:

(a) $4^3 = 64$ (b) $9^{\frac{1}{2}} = 3$ (c) $\sqrt[3]{8} = 2$ (d) $10^{-4} = 0.0001$

E.g. 4 Rewrite in index form:

(a) $\log_3 81 = 4$ (b) $\log_{125} 5 = \frac{1}{3}$ (c) $\log_2 \frac{1}{32} = -5$ (d) $\ln a = 3b$

E.g. 5 By writing the following in index notation, find the value of x:

(a) $\log_x 49 = 2$ (b) $\log_4 x = 3$ (c) $\log_x 7 = \frac{1}{3}$

Video: What do we mean by a logarithm?
Video: Natural logarithms

Solutions to Starter and E.g.s

Exercise
p117 7A Qu 1iace…(nc), 2iac, 4i, 5i, 6-12

Summary

$\log_a c \equiv$ to what power must a be raised to get c

$a^b = c$ $\Leftrightarrow$ $\log_a c = b$
"a b c" "log a c b"
$\text{Base}^{\text{Power}} = \text{Number}$ $\log_{\text{Base}} \text{Number} = \text{Power}$

Notation: $\ln x \equiv \log_e x$ — always use the $\ln$ notation
$\log x = \log_{10} x$ i.e. *if no base is written, assume it is base 10*