Optimisation

Starter

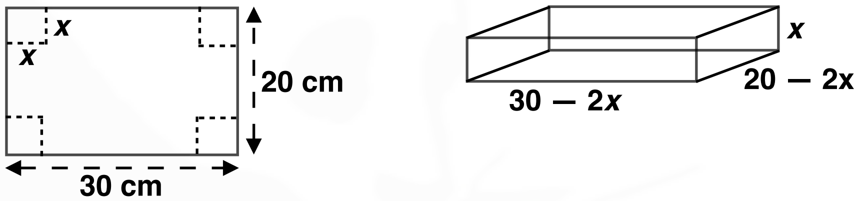
1. **(Review of last lesson)** Consider the curve given by $y = x^4 + kx^3 + x^2 + 17$.
   (a) Find the range of values of k if the curve has only 1 stationary point.
   (b) Find the coordinates of the stationary point and say whether it's a maximum or minimum.

Notes

Optimisation problems look at getting the maximum out of the raw material that are available.

E.g. 1 A open top box is made from a sheet of paper 30cm by 20cm by cutting out squares from the corners of the sheet of paper and folding the sides up. Find maximum volume of the box.

Working:



Let a square of side x be removed from each corner and let V be the volume of the open top box.

$V = x(30 - 2x)(20 - 2x)$
$\quad = 600x - 100x^2 + 4x^3$

$\frac{dV}{dx} = 600 - 200x + 12x^2$

A maximum occurs when $\frac{dV}{dx} = 0$ so $600 - 200x + 12x^2 = 0$

Solving the quadratic gives $x = 3.92$ or $x = 12.7$

$\frac{d^2V}{dx^2} = -200 + 24x$

When $x = 3.92$, $\frac{d^2V}{dx^2} < 0$ so $x = 3.92$ gives a maximum volume

When $x = 12.7$, $\frac{d^2V}{dx^2} > 0$ so $x = 12.7$ gives a minimum volume

When $x = 3.92$, $V = 3.92(30 - 2 \times 3.92)(20 - 2 \times 3.92) \approx 1056$

So the maximum volume of the box is about 1056 cm³.

N.B. In practical problems, it is usually possible to discard one of the values immediately. In the example above, $x < 10$ otherwise the box would disappear so $x \neq 12.7$. It is fine to use such arguments to discard values but proof that the other value gives a maximum (or minimum) is still needed.

Success Criteria — optimisation problems

1. Draw a diagram — this will help you form the equations.
2. Form one or two equations based on the information given.
3. Make sure the equation to be differentiated only has one unknown in it — substitute one equation into the other if necessary (see example below).
4. Differentiate.
5. Put the derivative equal to zero and solve.
6. Prove that one of the values gives the required maximum or minimum.
7. Substitute back into the original equation.

E.g. 2 A farmer has 80 m of fencing to mark out a rectangular enclosure, of dimensions x by y, against a brick wall. Find the maximum area that can be enclosed.

E.g. 3 A closed rectangular box is made of thin sheet metal and its length is three times its width.

(a) If the volume of the box is 288 cm³, show that its surface area is equal to $\left(\frac{768}{x} + 6x^2\right)$ cm², where x is the width of the box.

(b) Find by differentiation the dimensions of the box of least surface area.

Video: Applications of stationary points
Video: Maximising the volume of a box
Video: Minimising surface area of a box

Applications of stationary points EQ

Solutions to Starter and E.g.s

Exercise

p285 14C Qu 2, 4, 5 (S = …), 7, 10 (S = …)
Surface area of a cylinder $= 2\pi r^2 + 2\pi rh$

Summary

Optimisation problems:

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4. Differentiate.
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