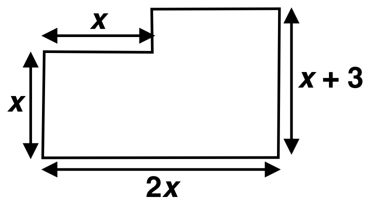
Quadratic Graphs

Starter

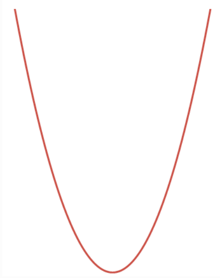
1. (Review of last lesson) The shape has an area of 44 units². Find the value of x.



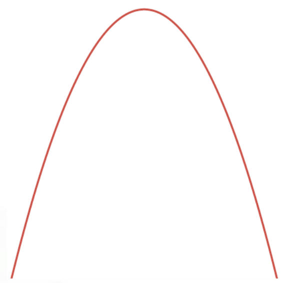
Notes

The general equation of a quadratic curve is given by $y = ax^2 + bc + c$. There are 2 basic shapes — concave-up or concave-down.

Concave-up (*minimum*)



Concave-down (*maximum*)



Sketching quadratic graphs

When sketching a graph, we avoid using a table of values and try to quickly get the main details. For quadratics, these are:

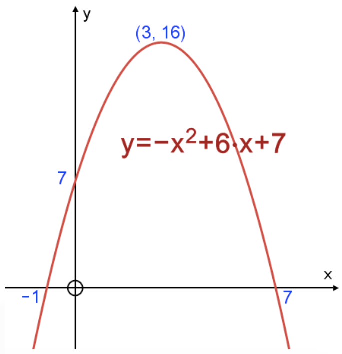
1. Solve "$= 0$" to find the **roots**: if $(x - a)$ is a factor $\Leftrightarrow x = a$ is root
2. ***Concave-up or concave-down***: $a > 0 \Rightarrow$ concave-up; $a < 0 \Rightarrow$ concave-down
3. $y-$**intercept** (i.e. when $x = 0$): c
4. **Turning point** (vertex): the $x-$coordinate is half-way between the roots
5. **Line of symmetry**: vertical line passing through the turning point

N.B. A sketch does not need a scale on the axes but ***important coordinates*** do need to be labelled
There is usually ***no need*** to draw the ***line of symmetry*** unless specifically asked

E.g. 1 Sketch the graph of $y = -x^2 + 6x + 7$, indicating the coordinates of axes intercepts and the turning point. Give the equation of the line of symmetry.

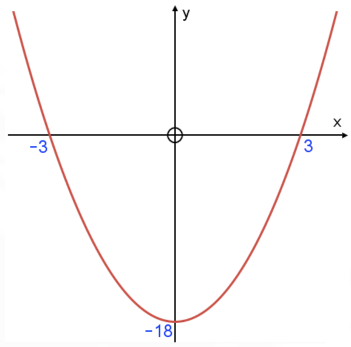
Working:

Roots: solve $7 + 6x - x^2 = 0$: $(1 + x)(7 - x) = 0$
$\therefore$ roots at $x = -1$ and $x = 7$
Concave-up or down: the coefficient of x^2 is $-1 < 0$, concave-down
$y-$**intercept**: when $x = 0$ so $y = 7$
Turning point (half way between the roots): i.e. $x = \dfrac{-1 + 7}{2} = 3$
When $x = 3$, $y = -3^2 + 6 \times 3 + 7 = 16$ so TP at $(3, 16)$
Line of symmetry: vertical line through the turning point $x = 3$



E.g. 2 Sketch the graphs of: (a) $y = x^2 - 6x + 8$ (b) $y = 2(3 - x)(1 + x)$
indicating the coordinates of axes intercepts and the turning point.
Give the equation of the line of symmetry.

E.g. 3 The sketch is a quadratic function of the form
$y = ax^2 + bx + x$. Find the value of a, b and c

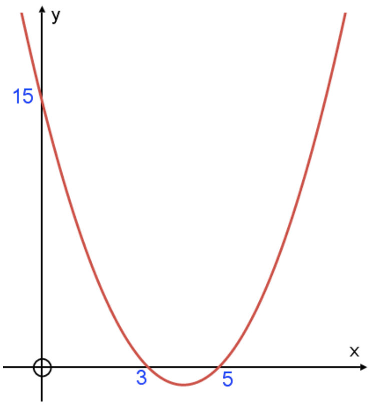


Working:

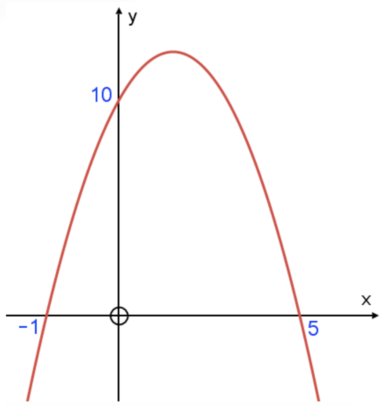
Root at $x = -3$ so $(x + 3)$ is a factor
Root at $x = 3$ so $(x - 3)$ is a factor
Equation is $y = k(x - 3)(x + 3)$
Curve passes through (0, −18): $-18 = k \times -3 \times 3 \quad \therefore k = 2$
Equation is $y = 2(x - 3)(x + 3)$
Expanding gives $y = 2x^2 - 18$
So $a = 2$, $b = 0$ and $c = -18$

E.g. 4 These sketches are graphs of quadratic functions of the form $y = ax^2 + bx + x$. Find the values of a, b and c for each function.

(a)



(b)



E.g. 5 The graph of $y = ax^2 + bc + c$ has a minimum at (5, −3) and passes through (4, 0). Find the values of a, b and c.

Video: Sketching quadratics

Solutions to Starter and E.g.s

Exercise
p33 3B Qu 1, 2i, 3

Summary
Sketching quadratic graphs:

1. Solve "$= 0$" to find the **roots**: if $(x - a)$ is a factor $\Leftrightarrow x = a$ is root
2. ***Concave-up or concave-down***: $a > 0 \Rightarrow$ concave-up; $a < 0 \Rightarrow$ concave-down
3. $y-$**intercept** (i.e. when $x = 0$): c
4. **Turning point** (vertex): the $x-$coordinate is half-way between the roots
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