Quadratic Inequalities

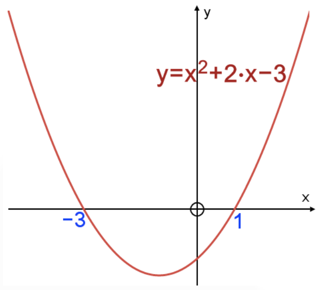
Starter

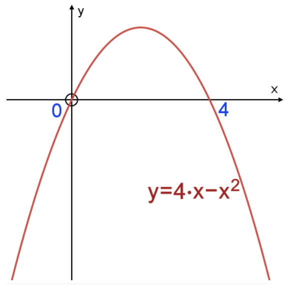
1. (Review of GCSE material) Solve the inequalities. Express your answers in set notation.

 (a) $\frac{43-3x}{7} < 4$ (b) $7 \leq 5x - 8 \leq 22$

2. Use the graphs given to solve the quadratic inequalities, expressing your answers in set notation:

 (a) $x^2 + 2x - 3 < 0$ (b) $4x - x^2 < 0$





Notes

Inequalities symbols: $<$ less than $\leq$ less than or equal to
$>$ greater than $\geq$ greater than or equal to

N.B. The "arrow" points to the smallest number.

When ***multiplying or dividing by a negative number***, ***change*** the ***direction*** of the ***inequality symbol***.

For single inequalities, the unknown is always written on the LHS i.e. $x \geq 5$, ***not*** $5 \leq x$

Compound inequality: small $< x <$ big
Smaller number is always on the left
Inequality symbols in compound inequalities ***always*** point to the left

Success Criteria: solving quadratic inequalities

1. If necessary, rearrange so that the coefficient of x^2 is positive and make sure the RHS is zero
2. Replace the inequality sign by an "=" sign and solve to find the roots
3. Sketch the graph of the original quadratic, using the roots as critical values and deciding whether it is concave-up or concave-down
4. Look at the original inequality — do we need the part above or below the $x-$axis?
 > 0 or ≥ 0 $\Rightarrow$ above the $x-$axis
 < 0 or ≤ 0 $\Rightarrow$ below the $x-$axis
5. Write the solution in set notation

E.g. 1 Solve the inequality $x^2 + x > 2$.

Working:

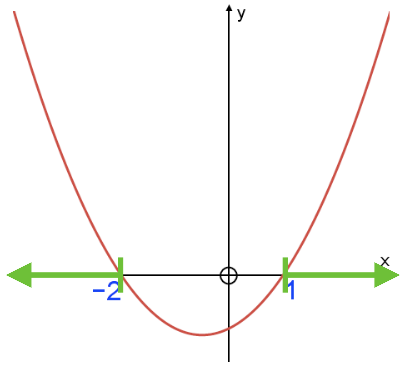
Rearrange $x^2 + x - 2 > 0$
Solve $x^2 + x - 2 = 0$ to find the roots:
$(x + 2)(x - 1) = 0$

Roots are $x = -1$ and $x = 2$

Coefficient of x^2 is +ve so concave-up
$> 0 \Rightarrow$ above the $x-$axis

We need the $x-$values **to the left of −2** and **to the right of 1**.

$\{x : x < -2\} \cup \{x : x > 1\}$



N.B. If $(x + 2)(x - 1) > 0$, it is wrong to write $x + 2 > 0$ and $x - 1 > 0$. Inatead, draw the graph to solve the inequality.

E.g. 2 Solve the inequalities: (a) $3x^2 + 5x - 2 < 0$ (b) $10x^2 \geq x + 3$
N.B. For (b), feel free to use your calculator to find the roots.

Unknown in denominator

When the unknown is in the denominator (e.g. $\frac{2}{x} < 1$), we cannot simply multiply by x. This is because x could be positive or negative. Multiplying by x^2 causes no ambiguity, since $x^2 \geq 0$.

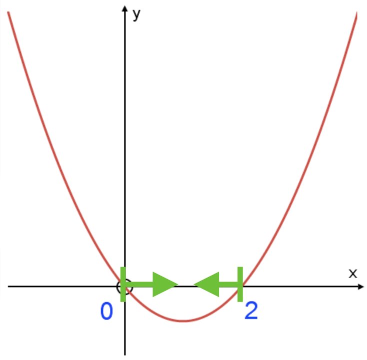
E.g. 3 Solve $\frac{2}{x} \geq 1$.

Working:

Multiplying by x^2: $2x \geq x^2$
Rearranging: $x^2 - 2x \leq 0$
Solving: $x(x - 2) \leq 0$
$\therefore$ the roots are $x = 0$ or $x = 2$

Coefficient of x^2 is +ve so concave-up
$\leq 0 \Rightarrow$ below the $x-$axis

We need the $x-$values **to the right of 0** and **to the left of 2**.
$\{x : x \geq 0\} \cap \{x : x \leq 2\}$



E.g. 4 Solve $\frac{1}{x} + 3 > 2$.

More than one inequality

When a question includes two inequalities, start by solving them separately. Then compare their solutions on a number line (using the circle and ball notation). The solution set is where both inequalities are valid.

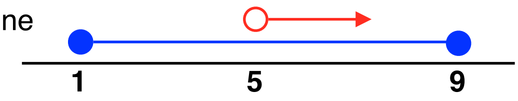
E.g. 5 State the set of values that satisfies $x > 5$ and $1 \leq x \leq 9$.

Working: Draw both inequalities on a number line

The solution is the overlap

$5 < x \leq 9$

$\{x : x > 5\} \cap \{x : x \leq 9\}$



E.g. 6 Find the values of x which satisfy both $4(3 - x) \geq 13 - 5x$ and $7x + 6 \geq 3x^2$.

E.g. 7 A rectangular office is to be built measuring $(x - 9)$ metres wide and $(x - 6)$ metres long. Given that at least 28 m^2 of floor space is required, find the set of possible values of x.

Video: Quadratic inequalities

Solutions to Starter and E.g.s

Exercise

p44 3D Qu 1iace, 2ibd, 3-5 (2ic = 3)

Summary

When ***multiplying or dividing by a negative number***, ***change*** the ***direction*** of the ***inequality symbol***.

Compound inequality: small $< x <$ big

Solving quadratic inequalities

1. If necessary, rearrange so that the coefficient of x^2 is positive and make sure the RHS is zero
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