Simplifying before Differentiating

Starter

1. **(Review of last lesson)**

Find the first derivative of: (a) $f(x) = 7x^{\frac{3}{8}}$ (b) $f(x) = -4x^{-\frac{12}{5}}$

2. Rewrite the following in the form kx^n:

(a) $\sqrt{x}$ (b) $\sqrt[3]{x^5}$ (c) $\frac{5}{x^7}$ (d) $\frac{8}{3\sqrt[4]{x^9}}$

Notes

Some functions require a simplification step before they can be differentiated.

With ***fractional coefficients*** (e.g. $\frac{2}{3}x^4$), make sure there is a numerator and denominator.

For example, it is better to write $\frac{2x^4}{3}$ than $\frac{2}{3}x^4$, because, when we differentiate, the 4 multiplies the numerator i.e. the 2. $\frac{dy}{dx} = \frac{2 \times 4x^3}{3} = \frac{8x^3}{3}$

If you wrote $\frac{2}{3}x^4$, you may misinterpret this as $\frac{2}{3x^4}$ if you are under pressure in an exam.

Common simplifications

Brackets: expand the brackets. ***E.g.*** $y = x(x+4) = x^2 + 4x$ $\frac{dy}{dx} = 2x + 4$

Roots: express in the form kx^n. ***E.g.*** $y = \sqrt[4]{x} = x^{\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4\sqrt[4]{x^3}}$

Reciprocal functions (negative powers): express in the form kx^n.

E.g. $y = \frac{3}{4x^6} = \frac{3x^{-6}}{4}$ $\frac{dy}{dx} = -\frac{18x^{-7}}{4} = -\frac{9}{2x^7}$

Fractions: form two separate fractions

E.g. $y = \frac{x^3+5}{x} = \frac{x^3}{x} + \frac{5}{x} = x^2 + 5x^{-1}$ $\frac{dy}{dx} = 2x - 5x^{-2} = 2x - \frac{5}{x^2}$

E.g. 1 Differentiate these functions:

(a) $y = \frac{1}{x}$ (b) $f(x) = -\frac{5}{x^7}$ (c) $y = \frac{9}{4x^5}$

Working: (a) $y = \frac{1}{x} = x^{-1}$

$$\frac{dy}{dx} = -1x^{-2} = -\frac{1}{x^2}$$

E.g. 2 Differentiate these functions:

(a) $y = \sqrt{x^3}$ (b) $f(x) = \sqrt[7]{x^4}$ (c) $y = \frac{8}{\sqrt{x^5}}$

Working: (a) $y = \sqrt{x^3} = x^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{3x^{\frac{1}{2}}}{2} = \frac{3\sqrt{x}}{2}$$

E.g. 3 Differentiate these functions:

(a) $y = (3x-5)^2$ (b) $y = \frac{x^3-4}{3x}$ (c) $y = \frac{9x^5-7}{x^3}$

Working: (a)

(b) $y = \frac{x^3-4}{3x} = \frac{x^3}{3x} - \frac{4}{3x} = \frac{x^2}{3} - \frac{4x^{-1}}{3}$

$$\frac{dy}{dx} = \frac{2x}{3} + \frac{4x^{-2}}{3} = \frac{2x}{3} + \frac{4}{3x^2}$$

(c)

Video: Differentiating polynomials
Video: Differentiating polynomials EQ

Solutions to Starter and E.g.s

Exercise
p259 13D Qu 1i, 2i, 3i, 4-11

Summary
It is better to write $\frac{2x^4}{3}$ than $\frac{2}{3}x^4$.
Some functions require a simplification step before they can be differentiated.
Brackets: expand the brackets.
Roots: express in the form kx^n.
Reciprocal functions (negative powers): express in the form kx^n.
Fractions: form two separate fractions