Simplifying before Integrating

Starter

1. **(Review of last lesson)** Find: (a) $\int x^{\frac{3}{4}} dx$ (b) $\int x^{-\frac{2}{7}} dx$

Notes

In a similar way to differentiation, when a polynomial expression includes brackets, roots or reciprocals (i.e. negative powers) we must turn it into the form kx^n.

Common simplifications

Brackets: expand the brackets. ***E.g.*** $\int 3x(x-6)dx = \int (3x^2 - 18x)dx = x^3 - 9x^2 + c$

Roots: express in the form kx^n. ***E.g.*** $\int \sqrt[5]{x} dx = \int x^{\frac{1}{5}} dx = \frac{5}{6}x^{\frac{6}{5}} + c$

Reciprocal functions (negative powers): express in the form kx^n.

E.g. $\int \frac{5}{7x^3} dx = \int \frac{5x^{-3}}{7} dx = \frac{5x^{-2}}{7 \times -2} + c = -\frac{5}{14x^2} + c$

N.B. Notice that the 7 remains in the denominator during the transformation step.

Fractions: form two separate fractions

E.g.
$$\int \left(\frac{8x^5 + 9}{x^2}\right) dx = \int \left(\frac{8x^5}{x^2} + \frac{9}{x^2}\right) dx$$
$$= \int (8x^3 + 9x^{-2}) dx$$
$$= 2x^4 + \frac{9x^{-1}}{-1} + c$$
$$= 2x^4 - \frac{9}{x} + c$$

E.g. 1 Find: (a) $\int \frac{2}{x^3} dx$ (b) $\int \frac{1}{7x^5} dx$ (c) $\int 5\sqrt{x} dx$ (d) $\int 4x\sqrt{x} dx$

Working: (a) $\int \frac{2}{x^3} dx = \int 2x^{-3} dx = \frac{2x^{-2}}{-2} + c = -\frac{1}{x^2} + c$

E.g. 2 Find: (a) $\int 4x(x^2 - 1) dx$ (b) $\int \left(2x - \frac{1}{x}\right)^2 dx$

Working: (a) $\int 4x(x^2 - 1) dx = \int (4x^3 - 4x) dx = x^4 - 2x^2 + c$

E.g. 3 Find: (a) $\displaystyle\int \frac{3x^2}{\sqrt{x}}\,dx$ (b) $\displaystyle\int \left(\frac{2+x}{x^5}\right)dx$

Working: (a) $\displaystyle\int \frac{3x^2}{\sqrt{x}}\,dx = \int 3x^{\frac{3}{2}}\,dx = \frac{6}{5}x^{\frac{5}{2}} + c = \frac{6}{5}\sqrt{x^5} + c$

Video: Integrating polynomials

Integrating polynomials EQ

Solutions to Starter and E.g.s

Exercise
p296 15B Qn 1i, 2i, 3ii, 4-13

Summary

Make sure the integral is of the form $\int kx^n\,dx$ before integrating.