

## Solving Exponential Equations

### Starter

- (Review of last lesson)** Write these as a single logarithm:  
(a)  $5 \log_a 2 - 2 \log_a 4 + 3 \log_a 3$       (b)  $2 \log_a 2 - (\log_a 5 + \log_a 8)$
- (Review of last lesson)** Given that  $a$  and  $b$  are positive constants, and  $a > b$ , solve the simultaneous equations:  $a + b = 13$       and       $\log_6 a + \log_6 b = 2$ .

### Notes

Back to one of the first question from this topic:      Solve  $2^x = 9$ .

There are two possible methods — the first one may seem longer but it works even when questions become complicated.

#### **Method 1 — Take logs of both sides**

Take logs of both sides:       $\log 2^x = \log 9$       *literally write log on front of each term*  
Use the 3rd law of logs:       $x \log 2 = \log 9$   
Make  $x$  the subject:       $x = \frac{\log 9}{\log 2} = 3.17$  (3 s.f.)

**N.B.** When taking logs of both sides there is no need to write the base of the logarithm.  
When asked to give the answer exactly here, it means in terms of logs i.e.  $\frac{\log 9}{\log 2}$ .  
We could also take  $\ln$  of both sides and the answer would be  $x = \frac{\ln 9}{\ln 2} = 3.17$

#### **Taking logs of both sides and the Classwiz calculator**

The problem with the Classwiz calculator is that the base of the logarithm must be entered each time. Therefore, it is less time-consuming to use the  $\ln$  button rather than the  $\log$  button.

#### **Method 2 — Convert from index to log form**

Rewrite  $2^x = 9$  in log form:       $x = \log_2 9 = 3.17$  (3 s.f.)

**N.B.** Years ago, calculators only had base 10 and base  $e$  so method 2 was not available. It is recommended to use method 1, except in the simplest questions (which probably wouldn't appear in an examination),

**E.g. 1** Solve (a)  $7^x = 2$       (b)  $2^{3x-1} = 5$       (c)  $11^{6x} = 10^{90}$   
Give your answer exactly (i.e. in logs in its simplest form) and to 3 s.f.

**E.g. 2** Solve  $8 \times 5^x = 2$ , giving your answer exactly and to 3 s.f.

**Working:**  $8 \times 5^x = 2 \Rightarrow 5^x = \frac{1}{4}$

Take logs of both sides:  $\log 5^x = \log \frac{1}{4}$

Use 3rd law of logs:  $x \log 5 = \log \frac{1}{4}$

Rearrange:  $x = \frac{\log \frac{1}{4}}{\log 5} = -0.861$

**E.g. 3** Solve  $6 \times 7^{2x-1} = 23$ , giving your answer to 3 s.f..

**E.g. 4** Solve  $11 \times 7^{x+1} = 3^{x+2}$ , giving your answer exactly and to 3 s.f..

[Video: Exponential and log equations](#)  
[Video: Solving inequalities involving logs](#)

[Solutions to Starter and E.g.s](#)

### Exercise

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### Summary

Take logs of both sides — better to take  $\ln$  of both sides with the Classwiz calculator.