Using identities to solve equations

Starter

1. *(Review of last lesson)* Solve the equation $4\cos^2\theta - 7\cos\theta = 2$ for $0° \le \theta \le 360°$.

2. Solve $3\cos\theta\sin\theta = 5\cos\theta$ for $-180° \le \theta \le 180°$.

3. Using a suitable trigonometric identity, solve $4\sin\theta = \cos\theta$ for $0° \le \theta \le 360°$.

4. Using a suitable trigonometric identity, solve $2\sin^2\theta + 3\cos\theta - 3 = 0$ for $0° \le \theta \le 360°$.

Notes

Trigonometric equations often require an identity to solve them.

Solving trigonometric equations requiring $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$

When equations involve simple multiples of sin and cos with no other functions or numbers, rearrange to get $\dfrac{\sin\theta}{\cos\theta}$ and then replace by $\tan\theta$.

Use $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$:

When: simple multiples of sin and cos with ***no other functions or numbers***

How: ***rearrange*** to get $\dfrac{\sin\theta}{\cos\theta}$ and then replace by $\tan\theta$.

E.g. 1 Solve these equations for $-180° \le \theta \le 180°$ giving your answer to a suitable accuracy:

(a) $8\cos\theta = -3\sin\theta$ (b) $5\sin\theta - 7\cos\theta = 0$

Working: (a) $8\cos\theta = -3\sin\theta \Rightarrow \dfrac{\sin\theta}{\cos\theta} = -\dfrac{8}{3} \Rightarrow \tan\theta = -\dfrac{8}{3}$

Ignore the −ve sign

$\tan^{-1}\dfrac{8}{3} = 69.4°$

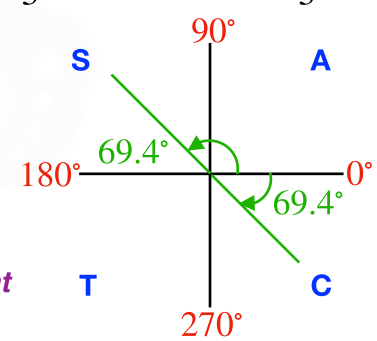
$-\dfrac{8}{3}$ is −ve

tan is −ve in the **S** & **C** quadrants

Draw the angle from the horizontal

Measure anti-clockwise for S quadrant

Measure clockwise for C quadrant



$\theta = 180° - 69.4°$ or $-69.4°$

$\theta = 111°$ or $-69.4°$ (3 s.f.)

Solving trigonometric equations requiring $\sin^2\theta + \cos^2\theta \equiv 1$

When a quadratic equation includes a quadratic term in $\sin$ or $\cos$ and a simple term in $\cos$ or $\sin$, replace the quadratic term using the identities:

$$\cos^2\theta \equiv 1 - \sin^2\theta$$
$$\text{or } \sin^2\theta \equiv 1 - \cos^2\theta$$

Use $\cos^2\theta \equiv 1 - \sin^2\theta$ or $\sin^2\theta \equiv 1 - \cos^2\theta$:

When: a ***quadratic*** equation includes a term in $\sin^2$ or $\cos^2$ and a simple multiple of $\cos$ or $\sin$.

How: ***replace*** the ***quadratic term***..

E.g. 2 Solve these equations for $-180° \leq \theta \leq 180°$ giving your answer to a suitable accuracy:

(a) $6\cos^2\theta + \sin\theta - 5 = 0$ (b) $3\cos\theta + 7 = 9\sin^2\theta$

Working: (a) $6\cos^2\theta + \sin\theta - 5 = 0 \Rightarrow 6(1 - \sin^2\theta) + \sin\theta - 5 = 0$

$$6\sin^2\theta - \sin\theta - 1 = 0$$
$$(3\sin\theta + 1)(2\sin\theta - 1) = 0$$
$$\sin\theta = \frac{1}{2} \text{ or } \sin\theta = -\frac{1}{3}$$

$\sin\theta = \frac{1}{2}$

$\sin^{-1}\frac{1}{2} \approx 30°$

$\frac{1}{2}$ is $+$ve

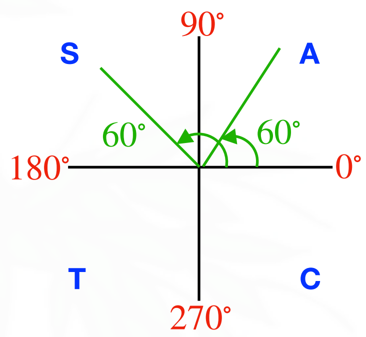
$\sin$ is $+$ve in the **A** & **S** quadrants

Draw the angle from the horizontal

Measure anti-clockwise from the $+$ve $x-$axis

$\theta = 30°$ or $180° - 30°$

$\theta = 30°$ or $150°$



$\sin\theta = -\frac{1}{3}$

Ignore the negative sign

$\sin^{-1}\frac{1}{3} \approx 19.47°$

$-\frac{1}{3}$ is $-$ve $\Rightarrow$ **T** & **C** quadrants

Draw the angle from the horizontal

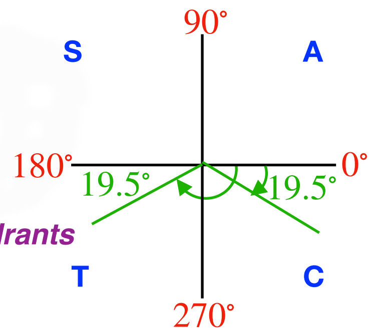
Measure anti-clockwise for S & T quadrants

$\theta = -19.47°$ or $-180° + 19.47°$

$\theta = -19.5°$ or $-161°$

The required angles are

$-161°, -19.5°, 30°$ or $150°$



Video: Solving equations using identities

Exam questions: Trigonometric equations

Solutions to Starter and E.g.s

Exercise
p199 10H Qu 1i, 2i, 3-12

Summary
Solving trigonometric equations involving identities

Use $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$:

When: simple multiples of $\sin$ and $\cos$ with ***no other functions or numbers***

How: ***rearrange*** to get $\dfrac{\sin\theta}{\cos\theta}$ and then replace by $\tan\theta$.

Use $\cos^2\theta \equiv 1 - \sin^2\theta$ or $\sin^2\theta \equiv 1 - \cos^2\theta$:

When: a ***quadratic*** equation includes a term in $\sin^2$ or $\cos^2$ and a simple multiple of $\cos$ or $\sin$.

How: ***replace*** the ***quadratic term***..