

Hypothesis Testing with Binomial distribution MS [70]

1.

(a)	States both hypotheses correctly for one-tailed test	AO2.5	B1	X = number of Christmas holidays without illness since January 2007 $X \sim B(7, p)$ $H_0 \quad p = 0.65$ $H_1 \quad p < 0.65$
	States model used (condone 0.009 rather than 0.056) PI	AO1.1b	M1	Under null hypothesis, $X \sim B(7, 0.65)$
	Using calculator, 0.056 or better	AO1.1b	A1	$P(X \leq 2) = 0.0556$
	Evaluates binomial model by comparing $P(X \leq 2)$ with 0.05 PI	AO3.5a	M1	$0.0556 > 0.05$
	Infers H_0 accepted PI	AO2.2b	A1	Accept H_0
	Concludes correctly in context. 'not sufficient evidence' or equivalent required	AO3.2a	E1	There is not sufficient evidence that the John's rate of illness has decreased

2.

(i)	Find $P(R \geq 6)$ or $P(R < 6)$ = 0.0083 or 0.9917 Compare with 0.025 [can be from N] [0.05 if "empty LH tail stated] Reject H_0	M1 A1 B1 A1√	4	Find $P(= 6)$ from tables/calc, OR RH critical region $P(\geq 6)$ in range [0.008, 0.0083] or $P(< 6) = 0.9917$ OR CR is 6 with probability 0.0083/0.9917 Explicitly compare with 0.025 [or 0.975 if consistent] OR state that result is in critical region Correct comparison and conclusion, √ on their p
(ii)	$n = 9, P(\leq 1) = 0.0385$ [> 0.025] $n = 10, P(\leq 1) = 0.0233$ [< 0.025] Therefore $n = 9$	M1 A1 B1	3	At least one, or $n = 8, P(\leq 1) = 0.0632$ Both of these probabilities seen, don't need 0.025 Answer $n = 9$ only, indep't of M1A1, <i>not</i> from $P(= 1)$

3.

(i)	$H_0: p = 0.65$ OR $p \geq 0.65$ $H_1: p < 0.65$ $B(12, 0.65)$	B2 M1	Both hypotheses correctly stated, in this form [One error (but not r , x or \bar{x}): B1] $B(12, 0.65)$ stated or implied
	$\alpha: P(\leq 6) = 0.2127$ Compare 0.10	A1 B1	Correct probability from tables, <i>not</i> $P(= 6)$ Explicit comparison with 0.10
	$\beta: \text{Critical region } \leq 5; 6 > 5$ Probability 0.0846	B1 A1	Critical region ≤ 5 or ≤ 6 or $\{\leq 4\} \cap \{\geq 11\}$ & compare 6 Correct probability
	Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65%	M1✓ A1✓ 7	Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. "consistent with claim" [SR: $N(7.8, 2.73)$: can get B2M1A0B1M0: 4 ex 7]
(ii)	Insufficient evidence to reject claim; test and p/q symmetric	B1✓ B1 2	Same conclusion as for part (i), don't need context Valid relevant reason, e.g. "same as (i)"
(iii)	$R \sim B(2n, 0.65), P(R \leq n) > 0.15$ $B(18, 0.65), p = 0.1391$ Therefore $n = 9$	M1 A1 A1 A1 4	$B(2n, 0.65), P(R \leq n) > 0.15$ stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR $< n$: M1A0, $n = 4, 0.1061$ A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR $B(12, 0.35)$: M1A0, $n = 4, 0.1061$ A1A0] 3 0.3529 7 0.1836 12 0.0942 4 0.2936 8 0.1594 13 0.0832 5 0.2485 9 0.1391 14 0.0736 6 0.2127 10 0.1218 15 0.0652

4.

$H_0: p = 0.4$ or $\mu = 4.8$ $H_1: p > 0.4$ or $\mu > 4.8$ $B(12, 0.4)$ $P(\geq 9) = 1 - 0.9847 = 0.0153$ < 0.05 Reject H_0 [can be implied] Significant evidence of increase in proportion of audience members who know sponsor's name	B2 M1 A1 B1✓ M1 A1✓ 7	Both fully correct, B2. [SR: one error, B1, but x or r or \bar{x} : B0] $B(12, 0.4)$ stated or implied, e.g. 0.9972 or 0.9847 Or: CR is ≥ 9 and $p \in [0.015, 0.0153]$ Explicitly compare with 0.05, or 9 with ≥ 9 , ✓ on $<$ Reject H_0 , ✓ on probability, must be " \geq " Conclusion interpreted in context [SR: $P(\leq 9)$ or $P(= 9)$ or $P(> 9)$: (B2 M1) A0 B1 M0A0] [SR: $N(4.8, 2.88)$: (B2) M1 A0 B0 M0A0]
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5.

(i)	$H_0: p = 0.14$ $H_1: p < 0.14$ $B(22, 0.14)$ $P(\leq 2) = .86^{22} + (22 \times .86^{21} \times .14) + (231 \times .86^{20} \times .14^2) = 0.3877$ > 0.1 Do not reject H_0 . Insufficient evidence that company overestimates viewing proportion	B2 M1 A1 A1 B1 M1 A1 8	Both correct. 1 error, B1, but x or r or \bar{x} etc: 0 $B(22, 0.14)$ stated or implied, e.g. $N(3.08, 2.6488)$ or $Po(3.08)$ Correct formula for 2 or 3 terms, or $P(\leq 0) = 0.036$ and CR Correct answer, a.r.t. 0.388, or CR is = 0 Explicitly compare 0.1 or CR with 2, OK from Po but <i>not</i> from N Correct comparison type and conclusion, needs binomial, at least 2 terms, <i>not</i> from $P(< 2)$ Contextualised, some acknowledgement of uncertainty [SR: Normal: B2 M1 A0 B0 M0] [SR: 2-tailed, or $p > 0.14, P(\geq 2)$: B1M1A2B0M1A1]
(ii)	Selected independently Each adult equally likely to be chosen	B1 B1 2	Independent selection Choice of sample elements equally likely (no credit if not focussed on selection) [Only "All samples of size n equally likely": B1 only unless related to Binomial conditions]

6.

(i)	B(14, 0.7) CR is ≥ 13 with probability 0.0475	M1 A1 A1 3	B(14, 0.7) stated or implied, e.g. N(9.8, 2.94), can be recovered CV 13, or > 12 or {13, 14}, allow = but no other inequalities Exactly correct CR, and supporting prob .0475 or .9525 seen
(ii)	$H_0: p = 0.7, H_1: p > 0.7$ 12 < 13 Do not reject H_0 . Insufficient evidence that proportion who show improvement is greater than 0.7	B2 B1 M1 A1 FT 5	Both, B2. Allow π . One error, B1, but r, x etc: B0 Compare CV from correct tail and inequality with 12, or $P(\geq 12) = 0.1608$ and > 0.05 or $P(< 12) = 0.8392$ and < 0.95 Correct method & conclusion, requires like-with-like; CV method needs ≥ 13 or < 12 ; p method needs ≥ 12 or < 12 Withhold if inconsistent Contextualised, acknowledge uncertainty [SR: Normal or Po: (i) M1, (ii) B2 maximum] [0.9932 or 0.0068 probably B2 maximum]

7.

(i)	$H_0: p = 0.4; H_1: p < 0.4$ B(10, 0.4)	B1B1 M1	Both: B2. Allow π . One error, B1, but x or r : 0. <i>SEE NOTES AT START AND END</i> B(10, 0.4) stated or implied, e.g. N(4, 2.4) [$P(=1)$ [=0.0404] or $P(\geq 1)$ [=0.9940] or $P(< 1)$ [=0.0060] or Poisson or normal, or RH tail for CR, gets no more marks in (i)]
α	$P(\leq 1) = 0.0464$ < 0.05 so reject H_0	A1 A1	This probability or 0.9536 only Explicit comparison with 0.05, or 0.9536 with 0.95
β	CR is ≤ 1 and compare 1 Probability of this is 0.0464	A1 A1	Comparison needn't be explicit in this method This probability needs to be seen
	Reject H_0 . Significant evidence that % who book with travel agents reduced	M1 A1 \checkmark 7	Correct method, \checkmark , comparison and first conclusion Interpreted in context, "evidence that" or equiv needed, \checkmark on numbers
(ii)	Can't deduce cause-and-effect	B1 1	Equivalent comment, regardless of answer to (i). Ignore wrong answer if right answer seen "Other factors haven't been considered" B1 "Sample is small" or "test may be wrong" B0

8.

α :	$H_0: p = \frac{1}{2}$ [or 0.33 or better] $H_1: p \neq \frac{1}{2}$ [or 0.33 or better] B(12, $\frac{1}{2}$) stated or implied $P(\geq 7) = 1 - 0.9336 = 0.0664$ > 0.025	B2 M1 A1 A1	Allow π , but $\mu = \frac{1}{2}$ etc B1. Any other letter, B0 One-tailed, or no symbol, B1 max B(12, $\frac{1}{2}$) stated or implied, allow for N(4,8/3), Po(4) Probability in range [0.066, 0.067] Explicit comparison with 0.025, or $2p$ with 0.05	Not $\mu = 4$ (if in doubt, consult) If N used, or $P(\leq 7)$ or $P(= 7)$, no more marks
β :	CR is ≥ 8 , 7 not in CR Probability is 0.0188	A1 A1	Needs explicit comparison of 7 with CV Must be ≥ 7 , 0.019 or 0.0188 or better, allow 0.9812	1-tailed: A0 here regardless of value Need to be clear that CR is being used – look for comparison with 7. See also ζ .
	Do not reject H_0 . Insufficient evidence that statement is false.	M1 A1 \checkmark [7]	Needs correct method, including like-with-like, correct tail, ≥ 7 (or ≤ 6). If CV, needs right tail A1 needs "evidence" or equivalent. "Statement" is enough context here	Allow from 1-tail. 0.9812 or 0.0188 or 0.0476: M0 unless " ≥ 7 " stated or clearly using β . \checkmark on their p /CR. Withhold if answer refers only to p .