Topic X1: Indices, surds and quadratics (Pre-TT A) [50]

1.

(i) Express $\frac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$, where a and b are positive integers. [3]

(ii) Express $\sqrt{18} - \sqrt{2}$ in simplified surd form. [2]

(Total 5 marks)

2.

Express each of the following in the form 4^n:

(i) $\frac{1}{16}$, [1]

(ii) 64, [1]

(iii) 8. [2]

(Total 4 marks)

3.

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

(Total 4 marks)

4.

Simplify

(i) $\left(\sqrt[3]{x}\right)^6$, [1]

(ii) $\frac{3y^4 \times (10y)^3}{2y^5}$. [3]

(Total 4 marks)

5.

Solve the equation $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 10 = 0$. [5]

(Total 5 marks)

6.

(i) Express $2x^2 - 24x + 80$ in the form $a(x-b)^2 + c$. [4]

(ii) State the equation of the line of symmetry of the curve $y = 2x^2 - 24x + 80$. [1]

(iii) State the equation of the tangent to the curve $y = 2x^2 - 24x + 80$ at its minimum point. [1]

(Total 6 marks)

7.

(i) Solve the equation $5 - 8x - x^2 = 0$, giving your answers in simplified surd form. [3]

(ii) Solve the inequality $5 - 8x - x^2 \leqslant 0$. [2]

(iii) Sketch the curve $y = (5 - 8x - x^2)(x + 4)$, giving the coordinates of the points where the curve crosses the coordinate axes. [5]

(Total 10 marks)

8.

By using the substitution $y = (x+2)^2$, find the real roots of the equation

$$(x+2)^4 + 5(x+2)^2 - 6 = 0.$$ [6]

(Total 6 marks)

9.

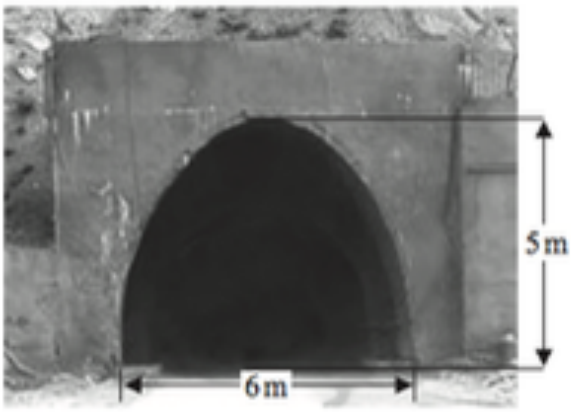

Figure 2

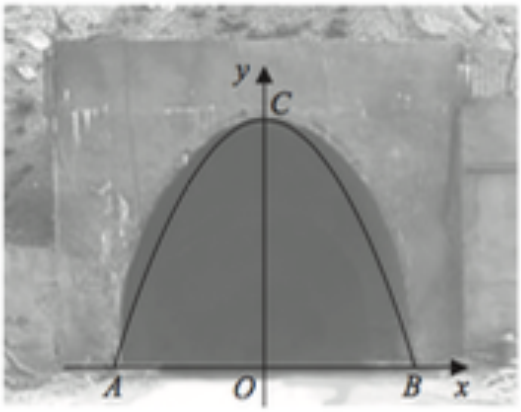

Figure 3

Figure 2 shows the entrance to a road tunnel. The maximum height of the tunnel is measured as 5 metres and the width of the base of the tunnel is measured as 6 metres.

Figure 3 shows a quadratic curve BCA used to model this entrance.

The points A, O, B and C are assumed to lie in the same vertical plane and the ground AOB is assumed to be horizontal.

(a) Find an equation for curve BCA.

(3)

A coach has height 4.1 m and width 2.4 m.

(b) Determine whether or not it is possible for the coach to enter the tunnel.

(2)

(c) Suggest a reason why this model may not be suitable to determine whether or not the coach can pass through the tunnel.

(1)

(Total 6 marks)