

## Topic X1: Indices, surds and quadratics (Pre-TT A) [50] MARKSCHEME

1.

$(i) \frac{12(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$ $= \frac{12(3-\sqrt{5})}{9-5}$ $= 9-3\sqrt{5}$	M1	Multiply numerator and denom by $3-\sqrt{5}$
	A1	$(3+\sqrt{5})(3-\sqrt{5}) = 9-5$
	A1	3
$(ii) 3\sqrt{2}-\sqrt{2}$ $= 2\sqrt{2}$	M1	Attempt to express $\sqrt{18}$ as $k\sqrt{2}$
	A1	$\frac{2}{5}$

2.

(i) $n = -2$	B1	
	$\frac{1}{5}$	
(ii) $n = 3$	B1	
	$\frac{1}{5}$	
(iii)	M1	$\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $(4^3)^{\frac{1}{2}}$ or $4 \times \sqrt{4}$ with brackets correct if used
$n = \frac{3}{2}$	A1	
	$\frac{2}{5}$	

3.

Question	Scheme	Marks	AOs
<b>10</b>	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$ ) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$ , which together with $k = 0$ gives $0 \leq k < \frac{3}{4}$ *	A1*	2.1
<b>(4 marks)</b>			
<b>Notes:</b>			
<b>B1:</b> Explains why $k = 0$ gives no real roots			
<b>M1:</b> Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark			
<b>M1:</b> Attempts solution of quadratic inequality			
<b>A1*:</b> Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)			

4.

2 (i)	$x^2$	B1	1	cao
(ii)	$\frac{3y^4 \times 1000y^3}{2y^5}$ $= 1500y^2$	B1		1000y <sup>3</sup> soi
		B1		1500
		B1	$\frac{3}{4}$	y <sup>2</sup>

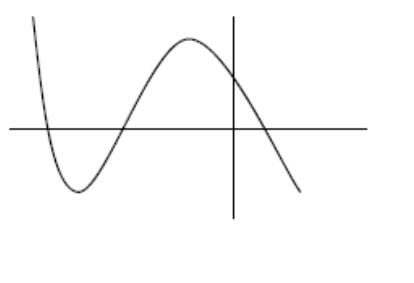
5.

4	<p>Let <math>y = x^{\frac{1}{3}}</math>  <math>y^2 + 3y - 10 = 0</math>  <math>(y - 2)(y + 5) = 0</math>  <math>y = 2, y = -5</math>  <math>x = 2^3, x = (-5)^3</math>  <math>x = 8, x = -125</math></p>	<p>*M1  DM1 A1 DM1 A1 ft 5  5</p>	<p>Attempt a substitution to obtain a quadratic or factorise with <math>\sqrt[3]{x}</math> in each bracket  Correct attempt to solve quadratic Both values correct Attempt cube Both answers correctly followed through  SR B2 <math>x = 8</math> from T &amp; I</p>
---	--	---	--

6.

6 (i)	<p><math>2(x^2 - 12x + 40)</math>  <math>= 2[(x - 6)^2 - 36 + 40]</math>  <math>= 2[(x - 6)^2 + 4]</math>  <math>= 2(x - 6)^2 + 8</math></p>	<p>B1 B1 M1 A1 4</p>	<p><math>a = 2</math>  <math>b = 6</math>  <math>80 - 2b^2</math> or <math>40 - b^2</math> or <math>80 - b^2</math> or <math>40 - 2b^2</math> (their <math>b</math>)  <math>c = 8</math></p>
(ii)	$x = 6$	B1 ft 1	
(iii)	$y = 8$	B1 ft 1	6

7.

8 (i)	<p><math>x = \frac{8 \pm \sqrt{(-8)^2 - (4 \times -1 \times 5)}}{-2}</math>  <math>= \frac{8 \pm \sqrt{84}}{-2}</math>  <math>= -4 - \sqrt{21}</math> or <math>= -4 + \sqrt{21}</math></p>	<p>M1 A1 A1 3</p>	<p>Correct method to solve quadratic  <math>x = \frac{8 \pm \sqrt{84}}{-2}</math>  Both roots correct and simplified</p>
(ii)	$x \leq -4 - \sqrt{21}, x \geq -4 + \sqrt{21}$	<p>M1 A1 2</p>	<p>Identifying <math>x \leq</math> their lower root, <math>x \geq</math> their higher root  <math>x \leq -4 - \sqrt{21}, x \geq -4 + \sqrt{21}</math>  (not wrapped, no 'and')</p>
(iii)		<p>B1 B1 B1 B1 B1 5</p>	<p>Roughly correct negative cubic with max and min  (-4, 0)  (0, 20)  Cubic with 3 distinct real roots  Completely correct graph</p>

8.

Let $y = (x + 2)^2$ $y^2 + 5y - 6 = 0$	B1	Substitute for $(x + 2)^2$ to get $y^2 + 5y - 6 (= 0)$
$(y + 6)(y - 1) = 0$	M1 A1	Correct method to find roots Both values for $y$ correct
$y = -6$ or $y = 1$	M1 A1 A1 6	Attempt to work out $x$ One correct value Second correct value and no extra real values
$(x + 2)^2 = 1$ $x = -1$ or $x = -3$	<b>6</b>	

9.

Question	Scheme	Marks	AOs
<b>6 (a)</b>	Attempts to use an appropriate model; e.g. $y = A(3-x)(3+x)$ or $y = A(9-x^2)$	M1	3.3
	e.g. $y = A(9-x^2)$ Substitutes $x = 0, y = 5 \Rightarrow 5 = A(9-0) \Rightarrow A = \frac{5}{9}$	M1	3.1b
	$y = \frac{5}{9}(9-x^2)$ or $y = \frac{5}{9}(3-x)(3+x), \{-3 \leq x \leq 3\}$	A1	1.1b
		<b>(3)</b>	
<b>(b)</b>	Substitutes $x = \frac{2.4}{2}$ into their $y = \frac{5}{9}(9-x^2)$	M1	3.4
	$y = \frac{5}{9}(9-x^2) = 4.2 > 4.1 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		<b>(2)</b>	
<b>(b)</b> <b>Alt 1</b>	$4.1 = \frac{5}{9}(9-x^2) \Rightarrow x = \frac{9\sqrt{2}}{10}$ , so maximum width = $2\left(\frac{9\sqrt{2}}{10}\right)$	M1	3.4
	$= 2.545... > 2.4 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		<b>(2)</b>	
<b>(c)</b>	E.g. <ul style="list-style-type: none"> <li>Coach needs to enter through the centre of the tunnel. This will only be possible if it is a one-way tunnel</li> <li>In real-life the road may be cambered (and not horizontal)</li> <li>The quadratic curve <math>BCA</math> is modelled for the entrance to the tunnel but we do not know if this curve is valid throughout the entire length of the tunnel</li> <li>There may be overhead lights in the tunnel which may block the path of the coach</li> </ul>	B1	3.5b
		<b>(1)</b>	
<b>(6 marks)</b>			