

## Topic X1: Indices, surds and quadratics (Pre-TT B) [42] MARKSCHEME

1.

(i)	$x^{\frac{1}{3}} = 2$ $x = 8$	B1	1	(allow embedded values throughout question 1) 8
(ii)	$10^t = 1$ $t = 0$	B1	1	0
(iii)	$(y^{-2})^2 = \frac{1}{81}$ $y^{-4} = \frac{1}{81}$ $y = \pm 3$	B1 B1	2	$y = 3$ $y = -3$

2.

7 (i)	$b^2 - 4ac$ (a) $36 - 9 \times 4 = 0$ (b) $100 - 48 = 52$ (c) $4 - 20 = -16$	M1		Uses $b^2 - 4ac$
		A1		1 correct
		A1	3	3 correct
				<b>SR</b> All 3 values correct but $\sqrt{\quad}$ used <b>B1</b>
(ii)	(a) Fig 3	B1		1 correct matching
	(b) Fig 2	B1		3 correct matchings
	(c) Fig 5			
	(a) 1 root, touches x-axis once, line of symmetry $x = -3$ or root $x = -3$	B1		1 correct comment relating roots to touching/crossing x-axis or about line of symmetry or vertex o.e. for one graph
	(b) 2 roots, meets x-axis twice, line of symmetry $x = 5$	B1	4	2 further correct comments about roots, line of symmetry o.e. for the other 2 graphs
	(c) No real roots, does not meet x-axis			
			7	

3.

$k = x^2$	M1*	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^2$
$4k^2 + 3k - 1 = 0$		
$(4k - 1)(k + 1) = 0$	M1 dep	Correct method to solve a quadratic
$k = \frac{1}{4}$ (or $k = -1$ )	A1	
$x = \pm \frac{1}{2}$	M1 A1	Attempt to square root to obtain $x = \pm \frac{1}{2}$ and no other values

4.

(i)	$2x^2 + 12x + 13 = 2(x^2 + 6x) + 13$ $= 2[(x+3)^2 - 9] + 13$ $= 2(x+3)^2 - 5$	B1 B1 M1	4	$a = 2$ $b = 3$ $13 - 2b^2$ or $13 - b^2$ or $\frac{13}{2} - b^2$ (their $b$ )
(ii)	$2(x+3)^2 - 5 = 0$ $(x+3)^2 = \frac{5}{2}$ $x = -3 \pm \sqrt{\frac{5}{2}}$	A1  M1  A1  A1		$c = -5$  Uses correct quadratic formula or completing square method $x = \frac{-12 \pm \sqrt{40}}{4}$ or $(x+3)^2 = \frac{5}{2}$ $x = -3 \pm \sqrt{\frac{5}{2}}$ or $-3 \pm \frac{1}{2}\sqrt{10}$

5.

$3x - 2y = k$ intersects $y = 2x^2 - 5$ at two distinct points		
Eliminate $y$ and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
$\{3x - 2(2x^2 - 5) = k \Rightarrow\} -4x^2 + 3x + 10 - k = 0$	A1	1.1b
$\{“b^2 - 4ac” > 0 \Rightarrow\} 3^2 - 4(-4)(10 - k) > 0$	dM1	2.1
$9 + 16(10 - k) > 0 \Rightarrow 169 - 16k > 0$		
Critical value obtained of $\frac{169}{16}$	B1	1.1b
$k < \frac{169}{16}$ <u>o.e.</u>	A1	1.1b
	(5)	
Eliminate $y$ and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
$y = 2\left(\frac{1}{3}(k+2y)\right)^2 - 5 \Rightarrow y = \frac{2}{9}(k^2 + 4ky + 4y^2) - 5$		
$8y^2 + (8k-9)y + 2k^2 - 45 = 0$	A1	1.1b
$\{“b^2 - 4ac” > 0 \Rightarrow\} (8k-9)^2 - 4(8)(2k^2 - 45) > 0$	dM1	2.1
$64k^2 - 144k + 81 - 64k^2 + 1440 > 0 \Rightarrow -144k + 1521 > 0$		
Critical value obtained of $\frac{169}{16}$	B1	1.1b
$k < \frac{169}{16}$ <u>o.e.</u>	A1	1.1b
	(5)	

6.

2(i)	<p>EITHER</p> $3(x^2 + 4x) + 7$ $3(x+2)^2 - 12 + 7$ $3(x+2)^2 - 5$ <p>OR</p> $3(x^2 + 2ax + a^2) + b$ $3x^2 + 6ax + 3a^2 + b$ $6a = 12$ $a = 2$ $3a^2 + b = 7$ $b = -5$			
(ii)	$x = -2$			
		M1	$a = \frac{12}{6 \text{ or } 2}$	
		A1	$a = 2$	
		M1	$7 - a^2$ or $7 - 3a^2$ or $\frac{7}{3} - a^2$ (their a)	
		A1	$b = -5$	
		B1 ft	$x = -2$	
				5

7.

(i)	Total profit (or $t$ ) is large when price (or $p$ ) is high	B1 [1]	3.5b		
(ii)	<p>Passes through (0, 0) and (12, 0)</p> <p>hence <math>t = kp(12 - p)</math></p> <p><math>k = 200</math></p>	B1	3.1b		
		B1	3.3	Or $t = 200p(12 - p)$ Or $t = 200(12p - p^2)$	
		[2]			
(iii)	<p><math>6400 = 200p(12 - p)</math> oe</p> <p><math>p^2 - 12x + 32 = 0</math></p> <p><math>p = 4, p = 8</math></p> <p><math>4 \leq p \leq 8</math></p> <p>Price must be between £4 and £8</p>	M1 A1FT A1FT A1 [4]	3.4 1.1 1.1 3.4	<p><math>6400 = (\text{their } k)p(12 - p)</math></p> <p>Any correct equation in form <math>ap^2 + bp + c = 0</math></p> <p>BC, but any method allowed</p> <p>Allow <math>4 &lt; p &lt; 8</math></p>	FT (ii) FT (ii)
(iv)	<p>E.g. <math>p = 0</math> implies giving book for free.</p> <p>Unrealistic. oe</p> <p>E.g. When <math>p = 0</math>, <math>t = 0</math>; but <math>t</math> should be negative as would make a loss. Unrealistic. oe</p> <p>E.g. When <math>p = 12.1</math>, <math>t</math> is negative. Possibly realistic as could make a loss if <math>p</math> set too high. oe</p>	E1 E1 [2]	3.2b 3.2b	<p>Valid comment about <math>p = 0</math></p> <p>Valid comment about <math>p = 12.1</math></p>	