

Topic X2: Logarithms exponentials and vectors (Post-TT) [43]

1.

(i) Express $\log_3(4x + 7) - \log_3 x$ as a single logarithm. [1]

(ii) Hence solve the equation $\log_3(4x + 7) - \log_3 x = 2$. [3]

(Total 4 marks)

2.

Use logarithms to solve the equation $3^{2x+1} = 5^{200}$, giving the value of x correct to 3 significant figures. [5]

(Total 5 marks)

3.

The mass, m grams, of a substance is increasing exponentially so that the mass at time t hours is given by

$$m = 250e^{0.021t}.$$

(i) Find the time taken for the mass to increase to twice its initial value, and deduce the time taken for the mass to increase to 8 times its initial value. [3]

(ii) Find the rate at which the mass is increasing at the instant when the mass is 400 grams. [3]

(Total 6 marks)

4.

(i) Evaluate $\log_5 15 + \log_5 20 - \log_5 12$. [3]

(ii) Given that $y = 3 \times 10^{2x}$, show that $x = a \log_{10}(by)$, where the values of the constants a and b are to be found. [4]

(Total 7 marks)

5.

(i) Sketch the curve $y = 2 \times 3^x$, stating the coordinates of any intersections with the axes. [3]

(ii) The curve $y = 2 \times 3^x$ intersects the curve $y = 8^x$ at the point P . Show that the x -coordinate of P may be written as

$$\frac{1}{3 - \log_2 3}. \quad [5]$$

(Total 8 marks)

6.

The points A , B and C have position vectors $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ respectively. M is the midpoint of BC .

(i) Find the position vector of the point D such that $\overline{BC} = \overline{AD}$. [3]

(ii) Find the magnitude of \overline{AM} . [3]

(Total 6 marks)

7.

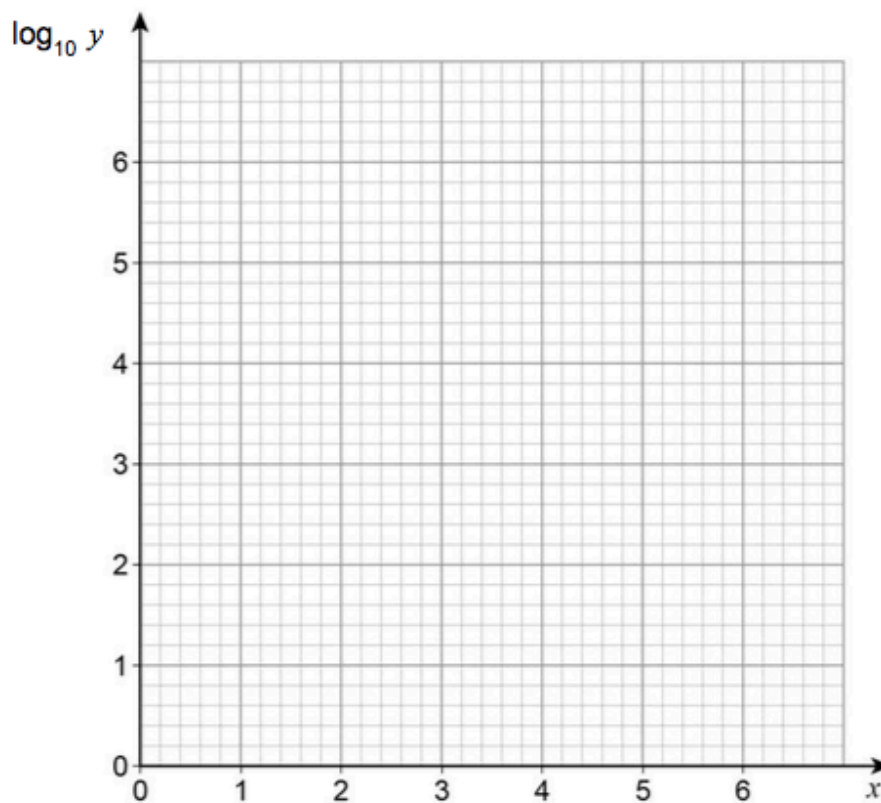
A student conducts an experiment and records the following data for two variables, x and y .

| | | | | | | |
|---------------|----|----|-----|------|------|------|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 14 | 45 | 130 | 1100 | 1300 | 3400 |
| $\log_{10} y$ | | | | | | |

The student is told that the relationship between x and y can be modelled by an equation of the form $y = kb^x$

- (a) Plot values of $\log_{10} y$ against x on the grid below.

[2 marks]



- (b) State, with a reason, which value of y is likely to have been recorded incorrectly.

[1 mark]

- (c) By drawing an appropriate straight line, find the values of k and b .

[4 marks]

(Total 7 marks)