Topic X2: Logarithms and exponentials (Post-TT) [43] MARKSCHEME

1.

(i)	$\log_3 \frac{4x+7}{x}$	B1	1	Correct single logarithm, as final answer, from correct working only
(ii)	$\log_3 \frac{4x+7}{x} = 2$			
	$\frac{4x+7}{x} = 9$	B1		State or imply $2 = \log_3 9$
	$4x + 7 = 9x$	M1		Attempt to solve equation of form $f(x) = 8$ or 9
	$x = 1.4$	A1	3	Obtain $x = 1.4$, or exact equiv

2.

3	$\log 3^{(2x+1)} = \log 5^{200}$	M1		Introduce logarithms throughout
	$(2x+1)\log 3 = 200\log 5$	M1		Drop power on at least one side
		A1		Obtain correct linear equation (now containing no powers)
	$2x + 1 = \frac{200\log 5}{\log 3}$	M1		Attempt solution of linear equation
	$x = 146$	A1	5	Obtain $x = 146$, or better
OR				
	$(2x + 1) = \log_3 5^{200}$	M1		Intoduce $\log_3$ on right-hand side
	$2x + 1 = 200\log_3 5$	M1		Drop power of 200
		A1		Obtain correct equation
		M1		Attempt solution of linear equation
		A1		Obtain $x = 146$, or better
			5	

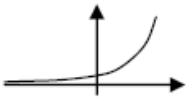
3.

(i)		Attempt process involving logarithm to solve $e^{0.021t} = 2$	M1	with t the only variable; at least as far as $0.021t = \ln 2$; must be …= 2
		Obtain 33	A1	or greater accuracy; ignore absence of, or wrong, units; final answer $\frac{\ln 2}{0.021}$ is A0
		State (or calculate separately to obtain) 99	B1√ [3]	following previous answer; no need to include units
(ii)		Differentiate to obtain $ke^{0.021t}$	M1	where k is any constant not equal to 250
		Obtain $250 \times 0.021 e^{0.021t}$	A1	or simplified equiv $5.25e^{0.021t}$
		Substitute to obtain 8.4 or $\frac{42}{5}$	A1 [3]	or value rounding to 8.4 with no obvious error

4.

(i)	$\log_5\left(\frac{15\times20}{12}\right)=\log_5 25=2$		M1 A1 A1	3	For any relevant combination of log a ± log b For log 25 – must follow correct working only For correct answer 2
(ii)	Method A	$\frac{1}{3}y=10^{2x}$ Hence $2x=\log_{10}\left(\frac{1}{3}y\right)$ i.e. $x=\frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	M1 M1 A1 A1	4	For correct division of both sides by 3 For relevant use of $a=b^c \Leftrightarrow c=\log_b a$ For correct equation involving logs to base 10 For correct answer for x
	Method B	$\frac{1}{3}y=10^{2x}$ $\log\frac{1}{3}y=\log 10^{2x}$ $\log\frac{1}{3}y=2x\log 10$ i.e. $x=\frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	M1 M1 A1 A1	4	For correct division of both sides by 3 For taking logs of both sides For correct linear equation involving logs For correct answer for x
	Method C	$y=3\times10^{2x}\Rightarrow\log y=\log 3\times10^{2x}$ $\log y=\log 3+\log 10^{2x}$ $\log y=\log 3+2x\log 10$ i.e. $x=\frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	M1 A1 M1 A1	4	For introducing logs throughout For correct RHS log 3 + log 10^{2x} For correct use of log $a^b = b$ log a For correct answer for x
	Method D	$x=a\log(b\times3\times10^{2x})$ $x=a\log 3b+a\log 10^{2x}$ $x=2ax\log 10\Rightarrow 2a=1\Rightarrow a=\frac{1}{2}$ $a\log 3b=0\Rightarrow 3b=1\Rightarrow b=\frac{1}{3}$	M1 M1 A1 A1	4 7	For substituting for y, and separating RHS into at least 2 terms For attempting values for a and b For obtaining $a = {}^1/_2$ For obtaining $b = {}^1/_3$

5.

(i)		**M1** **A1** **B1** **3**	Attempt sketch of exponential graph (1st quad) - if seen in 2nd quad must be approx correct Correct graph in both quadrants State or imply (0, 2) only
(ii)	$8^x = 2 \times 3^x$ $\log_2 8^x = \log_2 (2 \times 3^x)$	**M1**	Form equation in x and take logs (to any consistent base, or no base) – could use log 8
	$x\log_2 8 = \log_2 2 + x\log_2 3$	**M1** **M1**	Use log $a^b = b$ log a Use log ab = log a + log b ,or equiv with log a/b
	$3x = 1 + x\log_2 3$	**M1**	Use $\log_2 8 = 3$
	$x(3 - \log_2 3) = 1$, hence $x = \frac{1}{3-\log_2 3}$ **A.G.**	**A1**	Show given answer correctly
OR	$8^x = 2 \times 3^x$		
	$2^{3x} = 2 \times 3^x$ $2^{(3x-1)} = 3^x$ $\log_2 2^{(3x-1)} = \log_2 3^x$ $(3x-1)\log_2 2 = x\log_2 3$	**M1** **M1** **M1** **M1**	Use $8^x = 2^{3x}$ Attempt to rearrange equation to $2^k = 3^x$ Take logs (to any base) Use log $a^b = b$ log a
	$x(3 - \log_2 3) = 1$, hence $x = \frac{1}{3-\log_2 3}$ **A.G.**	**A1** **5**	Show given answer correctly

6.

(i)		$\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \mathbf{d} - \mathbf{a} = \overrightarrow{AD}$ $\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$	B1 M1 A1 [3]	1.1 3.1a 1.1	soi	
(ii)		$\overrightarrow{OM} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ $\lvert\overrightarrow{AM}\rvert = \sqrt{6^2 + 3^2} = 3\sqrt{5}$	B1 M1 A1 [3]	1.1 1.1 2.2a	soi Accept 6.71	

7.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Obtains (at least four) correct $\log_{10} y$ values, in table or plotted	AO1.1a	M1	(1, 1.1) (2, 1.7) (3, 2.1) (4, 3.0) (5, 3.1) (6, 3.5)
	Plots **all** points correctly	AO1.1b	A1	(Points above plotted on grid)
(b)	Identifies $y = 1100$ and gives correct reason	AO2.2b	B1	(4, 1100), as it is not on the line that the other points are close to
(c)	Uses laws of logs. (May earn in part **(a)** if laws of logs were used there)	AO1.1a	M1	$\log_{10} y = \log_{10} k + x \log_{10} b$ Vertical intercept c = 0.68 (=$\log_{10} k$) Therefore from intercept : $k = 10^{0.68}$
	Draws straight line **and** calculates/measures the vertical intercept c and attempts 10^c **or** calculates/measures gradient m and attempts 10^m Alternatively uses regression line from calculator to get intercept and gradient	AO1.1a	M1	Gradient m = 0.48 = $\log_{10} b$ Therefore from gradient : $b = 10^{0.48}$
	Finds correct value of b from 'their' gradient, provided 0.45< 'their' gradient < 0.51	AO1.1b	A1F	k = 4.8
	Finds correct value of k from 'their' intercept, provided 0.6 ≤ 'their' intercept ≤ 0.8	AO1.1b	A1F	b = 3.0
	Total		**7**	