

Topic X2: Logarithms exponentials and vectors (Pre-TT A) [43]

1.

Use logarithms to solve the equation $7^x = 2^{x+1}$, giving the value of x correct to 3 significant figures.

[5]

(Total 5 marks)

2.

Relative to a fixed origin O ,

the point A has position vector $\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$,
the point B has position vector $4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$,
and the point C has position vector $2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$.

Given that $ABCD$ is a parallelogram,

(a) find the position vector of point D .

(2)

The vector \vec{AX} has the same direction as \vec{AB} .

Given that $|\vec{AX}| = 10\sqrt{2}$,

(b) find the position vector of X .

(3)

(Total 5 marks)

3.

A cup of hot tea was placed on a table. At time t minutes after the cup was placed on the table, the temperature of the tea in the cup, θ °C, is modelled by the equation

$$\theta = 25 + Ae^{-0.03t}$$

where A is a constant.

The temperature of the tea was 75 °C when the cup was placed on the table.

(a) Find a complete equation for the model.

(1)

(b) Use the model to find the time taken for the tea to cool from 75 °C to 60 °C, giving your answer in minutes to one decimal place.

(2)

Two hours after the cup was placed on the table, the temperature of the tea was measured as 20.3 °C.

Using this information,

(c) evaluate the model, explaining your reasoning.

(1)

(Total 4 marks)

4.

The polynomial $f(x)$ is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) (a) Show that $(x + 1)$ is a factor of $f(x)$. [1]

(b) Hence find the exact roots of the equation $f(x) = 0$. [6]

(ii) (a) Show that the equation

$$2 \log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

can be written in the form $f(x) = 0$. [5]

(b) Explain why the equation

$$2 \log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

has only one real root and state the exact value of this root. [2]

(Total 14 marks)

5.

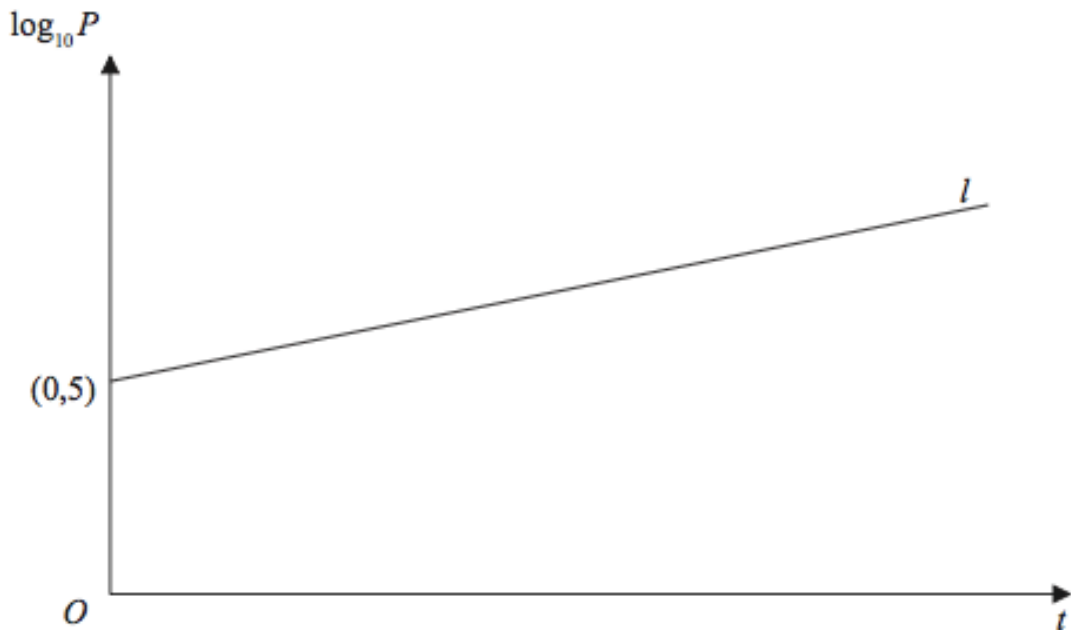


Figure 2

A town's population, P , is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at $(0, 5)$ as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l .

(2)

(b) Find the value of a and the value of b .

(4)

(c) With reference to the model interpret

- (i) the value of the constant a ,
- (ii) the value of the constant b .

(2)

(d) Find

- (i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,
- (ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

(Total 15 marks)