Topic X2: Logarithms and exponentials (Pre-TT A) [43] MARKSCHEME

1.

$\log 7^x = \log 2^{x+1}$	M1		Introduce logarithms throughout, or equiv with base 7 or 2
$x\log 7 = (x+1)\log 2$	M1		Drop power on at least one side
	A1		Obtain correct linear equation (allow with no brackets)
$x(\log 7 - \log 2) = \log 2$	M1		**Either** expand bracket and attempt to gather x terms, **or** deal correctly with algebraic fraction
$x = 0.553$	A1	5	Obtain $x = 0.55$, or rounding to this, with no errors seen
		[5]	

2.

Question	Scheme	Marks	AOs
4 (a)	$OA = \mathbf{i} + 7\mathbf{j} - 2\mathbf{k},\ OB = 4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k},\ OC = 2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$		
	$OD = OC + BA = (2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}) + (-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ or $OD = OA + BC = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + (-2\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OD} = -\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$	A1	1.1b
		(2)	
(b)	$\left\{\overrightarrow{AB} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} \Rightarrow\right\}\ \left\lvert\overrightarrow{AB}\right\rvert = \sqrt{(3)^2 + (-4)^2 + (5)^2}\ \left\{= \sqrt{50} = 5\sqrt{2}\right\}$	M1	1.1b
	As $\left\lvert\overrightarrow{AX}\right\rvert = 10\sqrt{2}$ then $\left\lvert\overrightarrow{AX}\right\rvert = 2\left\lvert\overrightarrow{AB}\right\rvert \Rightarrow \overrightarrow{AX} = 2\overrightarrow{AB}$		
	$\overrightarrow{OX} = \overrightarrow{OA} + 2\overrightarrow{AB} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + 2(3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ or $\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{AB} = (4 + 3\mathbf{j} + 3\mathbf{k}) + (3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OX} = 7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ only	A1	1.1b
		(3)	
		(5 marks)	

Question 4 Notes:

(a)	
M1:	A complete method for finding the position vector of *D*
A1:	$-\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} -1 \\ 14 \\ 4 \end{pmatrix}$
(b)	
M1:	A complete attempt to find $\left\lvert\overrightarrow{AB}\right\rvert$ or $\left\lvert\overrightarrow{BA}\right\rvert$
M1:	A complete process for finding the position vector of *X*
A1:	$7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ or $\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix}$

3.

3 (a)	$\{t=0, \theta=75 \Rightarrow 75 = 25 + A \Rightarrow A = 50\} \Rightarrow \theta = 25 + 50\mathrm{e}^{-0.03t}$	B1	3.3
		(1)	
(b)	$\{\theta = 60 \Rightarrow\} \Rightarrow 60 = 25 + "50"\mathrm{e}^{-0.03t} \Rightarrow \mathrm{e}^{-0.03t} = \frac{60-25}{"50"}$	M1	3.4
	$t = \frac{\ln(0.7)}{-0.03} = 11.8891648 = 11.9$ minutes (1 dp)	A1	1.1b
		(2)	
(c)	A valid evaluation of the model, which relates to the large values of *t*. E.g. • As $20.3 < 25$ then the model is not true for large values of *t* • $\mathrm{e}^{-0.03t} = \frac{20.3-25}{"50"} = -0.094$ does not have any solutions and so the model predicts that tea in the room will never be 20.3 °C. So the model does not work for large values of *t* • $t = 120 \Rightarrow \vartheta = 25 + 50e^{-0.03(120)} = 26.36...$ which is not approximately equal to 20.3, so the model is not true for large values of *t*	B1	3.5a
		(1)	
			(4 marks)

4.

(i) (a)	f(-1) = – 1 + 6 – 1 – 4 = 0	B1	1	Confirm f(-1) = 0, through any method
(b)	$x = -1$	B1		State x = -1 at any point
	$f(x) = (x+1)(x^2+5x-4)$	M1		Attempt complete division by (x + 1), or equiv
		A1		Obtain $x^2 + 5x + k$
		A1		Obtain completely correct quotient
	$x = \frac{-5 \pm \sqrt{25+16}}{2}$	M1		Attempt use of quadratic formula, or equiv, find roots
	$x = \frac{1}{2}\left(-5 \pm \sqrt{41}\right)$	A1	6	Obtain $\frac{1}{2}\left(-5 \pm \sqrt{41}\right)$
(ii) (a)	$\log_2(x+3)^2 + \log_2 x - \log_2(4x+2) = 1$	B1		State or imply that 2log (x + 3) = log (x + 3)^2
		M1		Add or subtract two, or more, of their algebraic logs correctly
	$\log_2\left(\frac{(x+3)^2 x}{4x+2}\right) = 1$	A1		Obtain correct equation (or any equivalent, with single term on each side)
	$\frac{(x+3)^2 x}{4x+2} = 2$	B1		Use $\log_2 a = 1 \Rightarrow a = 2$ at any point
	$(x^2+6x+9)x = 8x+4$			
	$x^3 + 6x^2 + x - 4 = 0$	A1	5	Confirm given equation correctly
(b)	$x > 0$, otherwise $\log_2 x$ is undefined	B1*		State or imply that log x only defined for $x > 0$
	$x = \frac{1}{2}\left(-5 + \sqrt{41}\right)$	B1√dep*	2	State $x = \frac{1}{2}\left(-5 + \sqrt{41}\right)$ (or x = 0.7) only, following their single positive root in (i)(b)
			14	

5.

Question	Scheme		Marks	AOs
14(a)	$\log_{10} P = mt + c$		M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$		A1	1.1b
			(2)	
(b)	**Way 1:** As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	**Way 2:** As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	$a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	So $a = 100\ 000$ **or** $b = 1.0116$		A1	1.1b
	Both $a = 100\ 000$ **and** $b = 1.0116$ (awrt 1.01)		A1	1.1b
			(4)	
(c)(i)	The initial population		B1	3.4
(c)(ii)	The proportional increase of population each year		B1	3.4
			(2)	
(d)(i)	300000 to nearest hundred thousand		B1	3.4
(d)(ii)	Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give $t =$		M1	3.4
	60.2 years to 3sf		A1ft	1.1b
			(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth		B2	3.5b
			(2)	