

Topic X2: Logarithms exponentials and vectors (Pre-TT B) [36]

1.

(a) Given that $\log_a x = p$ and $\log_a y = q$, express the following in terms of p and q .

(i) $\log_a(xy)$ [1]

(ii) $\log_a\left(\frac{a^2x^3}{y}\right)$ [3]

(b) (i) Express $\log_{10}(x^2 - 10) - \log_{10}x$ as a single logarithm. [1]

(ii) Hence solve the equation $\log_{10}(x^2 - 10) - \log_{10}x = 2 \log_{10}3$. [5]

(Total 10 marks)

2.

A substance is decaying in such a way that its mass, m kg, at a time t years from now is given by the formula

$$m = 240e^{-0.04t}.$$

(i) Find the time taken for the substance to halve its mass. [3]

(ii) Find the value of t for which the mass is decreasing at a rate of 2.1 kg per year. [4]

(Total 7 marks)

3.

(i) Sketch the curve $y = 6 \times 5^x$, stating the coordinates of any points of intersection with the axes. [3]

(ii) The point P on the curve $y = 9^x$ has y -coordinate equal to 150. Use logarithms to find the x -coordinate of P , correct to 3 significant figures. [3]

(iii) The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point Q . Show that the x -coordinate of Q can be written as $x = \frac{1 + \log_3 2}{2 - \log_3 5}$. [5]

(Total 11 marks)

4.

13. The growth of pond weed on the surface of a pond is being investigated.

The surface area of the pond covered by the weed, $A \text{ m}^2$, can be modelled by the equation

$$A = 0.2e^{0.3t}$$

where t is the number of days after the start of the investigation.

(a) State the surface area of the pond covered by the weed at the start of the investigation. (1)

(b) Find the rate of increase of the surface area of the pond covered by the weed, in m^2/day , exactly 5 days after the start of the investigation. (2)

Given that the pond has a surface area of 100 m^2 ,

(c) find, to the nearest hour, the time taken, according to the model, for the surface of the pond to be fully covered by the weed. (4)

The pond is observed for one month and by the end of the month 90% of the surface area of the pond was covered by the weed.

(d) Evaluate the model in light of this information, giving a reason for your answer. (1)

(Total 8 marks)