Topic X2: Logarithms exponentials and vectors (Pre-TT B) [36] MS

1.

(a)(i) $\log_a xy = p+q$	B1	1	State $p+q$ cwo
(ii) $\log_a\left(\frac{a^2x^3}{y}\right) = 2+3p-q$	M1		Use $\log a^b = b\log a$ correctly at least once
	M1		Use $\log\frac{a}{b} = \log a - \log b$ correctly
	A1	3	Obtain $2+3p-q$
(b)(i) $\log_{10}\frac{x^2-10}{x}$	B1	1	State $\log_{10}\frac{x^2-10}{x}$ (with or without base 10)
(ii) $\log_{10}\frac{x^2-10}{x} = \log_{10} 9$	B1		State or imply that $2\log_{10} 3 = \log_{10} 3^2$
$\frac{x^2-10}{x} = 9$	M1		Attempt correct method to remove logs
$x^2-9x-10=0$	A1		Obtain correct $x^2-9x-10=0$ aef, no fractions
$(x-10)(x+1)=0$	M1		Attempt to solve three term quadratic
$x=10$	A1	5	Obtain $x=10$ only
		10	

2.

(i) State $e^{-0.04t} = 0.5$	B1		or equiv
Attempt solution of equation of form $e^{-0.04t} = k$	M1		using sound process; maybe implied
Obtain 17	A1	3	or greater accuracy (17.328…)
(ii) Differentiate to obtain form $k e^{-0.04t}$	*M1		constant k different from 240
Obtain $(\pm)$ $9.6e^{-0.04t}$	A1		or (unsimplified) equiv
Equate attempt at first derivative to $(\pm)$ 2.1 and attempt solution	M1		dep *M; method maybe implied
Obtain 38	A1	4	or greater accuracy (37.9956…)

3.

(i) 	M1		Reasonable graph in both quadrants
	A1		Correct graph in both quadrants
	B1	3	State or imply (0, 6)
(ii) $9^x = 150$	M1		Introduce logarithms throughout, or equiv with $\log_9$
$x\log 9 = \log 150$	M1		Use $\log a^b = b\log a$ and attempt correct method to find x
$x = 2.28$	A1	3	Obtain $x = 2.28$
(iii) $6\times 5^x = 9^x$	M1		Form eqn in x and take logs throughout (any base)
$\log_3(6\times 5^x) = \log_3 9^x$	M1		Use $\log a^b = b\log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two
$\log_3 6 + x\log_3 5 = x\log_3 9$	M1		Use $\log ab = \log a + \log b$ correctly on $\log(6\times 5^x)$ or $\log 6$
$\log_3 3 + \log_3 2 + x\log_3 5 = 2x$	M1		Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)
$x(2-\log_3 5) = 1+\log_3 2$			
$x = \frac{1+\log_3 2}{2-\log_3 5}$ **A.G.**	A1	5	Obtain $x = \frac{1+\log_3 2}{2-\log_3 5}$ convincingly (inc base 3 throughout)
		11	

4.

Question	Scheme	Marks	AOs
13(a)	$0.2\ \text{m}^2$	B1	3.4
		(1)	
(b)	$A = 0.2\text{e}^{0.3t}$ Rate of change = gradient $= \dfrac{\text{d}A}{\text{d}t} = 0.06\text{e}^{0.3t}$	M1	3.1b
	At $t = 5 \Rightarrow$ Rate of Growth is $0.06\text{e}^{1.5} = 0.269$ m²/day	A1	1.1b
		(2)	
(c)	$100 = 0.2\text{e}^{0.3t} \Rightarrow \text{e}^{0.3t} = 500$	M1 A1	3.1a 1.1b
	$\Rightarrow t = \dfrac{\ln(500)}{0.3} = 20.7$ days 20 days 17 hours	M1 A1	1.1b 3.2a
		(4)	
	At $t = 5 \Rightarrow$ Rate of Growth is $0.06\text{e}^{1.5} = 0.269$ m²/day	A1	1.1b
		(2)	
(d)	The model given suggests that the pond is fully covered after 20 days 17 hours. Observed data is inconsistent with this as the pond is only 90% covered by the end of one month (28/29/30/31 days). Hence the model is not accurate	B1	3.5a
		(1)	
			(8 marks)

Notes:

(a)

B1: $0.2\ \text{m}^2$ oe

(b)

M1: Links rate of change to gradient and differentiates $0.2\text{e}^{0.3t} \rightarrow k\text{e}^{0.3t}$

A1: Correct answer 0.269 m²/day

(c)

M1: Substitutes $A = 100$ and proceeds to $\text{e}^{0.3t} = k$

A1: $\text{e}^{0.3t} = 500$

M1: Correct method when proceeding from $\text{e}^{0.3t} = k \Rightarrow t =$ **..**

A1: 20 days 17 hours

(d)

B1: Valid conclusion following through on their answer to (c).