

## Topic X3 Calculus (Post-TT A) [44]

1.

Find  $\frac{dy}{dx}$  in each of the following cases:

(i)  $y = 10x^{-5}$ , [2]

(ii)  $y = \sqrt[4]{x}$ , [3]

(iii)  $y = x(x+3)(1-5x)$ . [4]

(Total 9 marks)

2

(a) Given that  $f(x) = x^2 - 4x + 2$ , find  $f(3+h)$

Express your answer in the form  $h^2 + bh + c$ , where  $b$  and  $c \in \mathbb{Z}$ .

[2 marks]

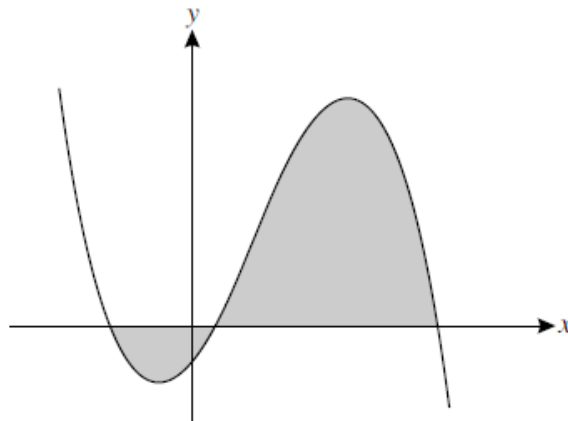
(b) The curve with equation  $y = x^2 - 4x + 2$  passes through the point  $P(3, -1)$  and the point  $Q$  where  $x = 3+h$ .

Using differentiation from first principles, find the gradient of the tangent to the curve at the point  $P$ .

[3 marks]

(Total 5 marks)

3.



The diagram shows the curve  $y = f(x)$ , where  $f(x) = -4x^3 + 9x^2 + 10x - 3$ .

(i) Verify that the curve crosses the  $x$ -axis at  $(3, 0)$  and hence state a factor of  $f(x)$ . [2]

(ii) Express  $f(x)$  as the product of a linear factor and a quadratic factor. [3]

(iii) Hence find the other two points of intersection of the curve with the  $x$ -axis. [2]

(iv) The region enclosed by the curve and the  $x$ -axis is shaded in the diagram. Use integration to find the total area of this region. [5]

(Total 11 marks)

4.

Find the equation of the normal to the curve  $y = x^3 - 4x^2 + 7$  at the point  $(2, -1)$ , giving your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers. [7]

(Total 7 marks)

5.

A curve has equation

$$y = 2x^3 - 2x^2 - 2x + 8$$

(a) Find  $\frac{dy}{dx}$

(2)

(b) Hence find the range of values of  $x$  for which  $y$  is increasing.  
Write your answer in set notation.

(4)

(Total 6 marks)

6.



The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is  $x$  metres.

(i) Show that the enclosed area,  $A \text{ m}^2$ , is given by

$$A = 20x - 2x^2. \quad [2]$$

(ii) Use differentiation to find the maximum value of  $A$ .

[4]

(Total 6 marks)