Topic X3 Calculus (Post-TT A) [44] MARKSCHEME

1.

(i) $\int(4x-5)\mathrm{d}x = 2x^2-5x+c$	M1		Obtain at least one correct term
	A1	2	Obtain at least $2x^2-5x$
(ii) $y = 2x^2 - 5x + c$	B1√		State or imply y = their integral from (i)
$7 = 2\times3^2-5\times3+c \Rightarrow c=4$	M1		Use (3,7) to evaluate c
So equation is $y = 2x^2-5x+4$	A1	3	Correct final equation
		5	

2.

Q	Marking Instructions	AO	Marks	Typical Solution
(a)	Substitutes $3+h$ to obtain a correct unsimplified expression for $\mathrm{f}(3+h)$	AO1.1a	M1	$(3+h)^2-4(3+h)+2$ or $=9+6h+h^2-12-4h+2$
	Expresses simplified answer correctly in given format	AO1.1b	A1	$=h^2+2h-1$
(b)	Identifies and uses $\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$ to obtain an expression for the gradient of chord Mark can be awarded for unsimplified expression.	AO1.1a	M1	Gradient of chord $=\dfrac{\mathrm{f}(3+h)-\mathrm{f}(3)}{h}$ $=\dfrac{h^2+2h-1+1}{h}$ $=h+2$ As $h\to0$, $h+2\to2$ Gradient of tangent $=2$
	Obtains a correct and full simplification	AO1.1b	A1	
	Deduces that, as h approaches 0 the limit of $\dfrac{\mathrm{f}(3+h)-\mathrm{f}(3)}{h}$ is 2 (Must not simply say $h=0$ but accept words rather than limit notation) FT 'their' gradient provided M1 has been awarded	AO2.2a	R1	
	Total		**5**	

3.

9(i)	$f(3) = -108 + 81 + 30 - 3 = 0$ hence $(x - 3)$ is a factor	**B1**		Show that $f(3) = 0$, detail required	Substitute $x = 3$ and confirm $f(3) = 0$ – must show detail of substitution rather than just state $f(3) = 0$. Allow $f(3) = -4 \times 3^3 + 9 \times 3^2 + 10 \times 3 - 3 = 0$ for B1.
		B1	2	State $(x - 3)$ as factor (allow $(3 - x)$ as the factor)	Not dependent on first B1. Must be seen in (i) so no back credit from (ii). Allow if not explicitly stated as factor (and allow $f(x) = x - 3$). Ignore other factors if also given at this stage.
(ii)	$f(x) = (x - 3)(-4x^2 - 3x + 1)$ or $f(x) = (3 - x)(4x^2 + 3x - 1)$ or $f(x) = (x + 1)(-4x^2 + 13x - 3)$ or $f(x) = (-x - 1)(4x^2 - 13x + 3)$ or $f(x) = (1 - 4x)(x^2 - 2x - 3)$ or $f(x) = (4x - 1)(-x^2 + 2x + 3)$	**M1**		Attempt complete division by $(x - 3)$, or equiv (allow division by $(3 - x)$)	Must be a full attempt to find three term quadratic. Can use inspection, but must be a reasonable attempt at middle term, with first and last correct. Can use coefficient matching, but must be full method with reasonable attempts at all 3 coefficients. Allow M1 if actually factorising $-f(x)$.
		A1		Obtain $-4x^2 - 3x + c$ or $-4x^2 + bx + 1$ (or the negative of these if dividing by $(3 - x)$)	c, b non-zero constants. First option is likely to come from division, second option from inspection. Coefficient matching could lead to either. Allow A1 for negative of either of these from factorising $-f(x)$.
		A1	3	Obtain $(x - 3)(-4x^2 - 3x + 1)$ (or $(3 - x)(4x^2 + 3x - 1)$)	Needs to be written as a product as per request in question paper. Allow $-(x - 3)(4x^2 + 3x - 1)$, but $(x - 3)(4x^2 + 3x - 1)$ is A0. A0 if now 3 linear factors and product of linear and quadratic never seen. If using one of the other two correct factors then all three marks are available, and apply mark scheme as above ie M1 for full attempt at division or equiv, A1 for lead term plus one other correct and A1 for product of linear and quadratic. **SR:** If candidates initially state three linear factors and then expand to get the product of a linear and quadratic as requested award **B3** if fully correct and simplified otherwise **B0**.
(iii)	$-4x^2 - 3x + 1 = 0$ $(1 - 4x)(x + 1) = 0$ $x = ¼$, $x = -1$	**M1**		Attempt to solve quadratic	If factorising, needs to give two correct terms when brackets expanded. If using formula allow sign slips only – need to substitute and attempt one further step. If completing the square must get to $(x + p) = \pm\sqrt{q}$, with reasonable attempts at p and q.
		A1	2	Obtain $(¼ , 0)$, $(-1, 0)$	Condone only x values given rather than coordinates. Allow if $x = 3$ is still present as well.
(iv)	$\int f(x)dx = -x^4 + 3x^3 + 5x^2 - 3x$	**B1**		Obtain $-x^4 + 3x^3 + 5x^2 - 3x$	Allow unsimplified coefficients. Condone $+ c$.
	$F(3) - F(¼) = (36) - (^{-101}/_{256}) = 36^{101}/_{256}$ $F(¼) - F(-1) = (^{-101}/_{256}) - (4) = -4^{101}/_{256}$	**M1***		Attempt $F(3) - F(¼)$ or $F(¼) - F(-1)$	Allow use of incorrect limits from their (iii). Limits need to be in correct order, and subtraction. Allow slips when evaluating but clear subtraction attempt must be seen or implied at least once. If minimal method shown then it must appear to be a plausible attempt eg $F(3) = 198$ or even $F(3) - F(¼) = 198.4$.
		A1		Obtain at least one correct area, including decimal equivs	Obtain $36^{101}/_{256}$ or $^{9317}/_{256}$ or 36.4 or $-4^{101}/_{256}$ or $-^{1125}/_{256}$ or -4.4 Can get A1 if both areas attempted and one is correct but the other isn't.
		M1d*		Attempt full method to find total area including dealing correctly with negative area	Need to see modulus of negative integral from attempt at $F(¼) - F(-1)$ (just changing sign from –ve to +ve is sufficient). If values incorrect in (iii) then can only get this mark if their integral gives negative value. Need to have positive integral from $F(3) - F(¼)$.
	Hence area $= 36^{101}/_{256} + 4^{101}/_{256} = 40^{101}/_{128}$	**A1**	5 12	Obtain $40^{101}/_{128}$ or $^{5221}/_{128}$ or 40.8	Allow exact fraction (including unsimplified ie $^{10442}/_{256}$), or decimal answer to 3dp or better (rounding to 40.8 with no errors seen) **SR:** If candidate attempts $F(3) - F(¼)$ **and** $F(-1) - F(¼)$ as an alternative method for dealing with negative area then mark as **B1** correct integral **M2** complete method **A1** obtain one correct area **A1** obtain correct total area Any attempts using this method must be fully supported by evidence of intention, especially -1 as top limit and ¼ as bottom limit used consistently throughout integration attempt. It should not be awarded if candidate appears to have simply confused their order of subtraction.

4.

$\frac{dy}{dx} = 3x^2 - 8x$	M1		Attempt to differentiate (one of $3x^2, -8x$)
	A1		Correct derivative
When $x = 2$, $\frac{dy}{dx} = -4$	M1 A1		Substitutes $x = 2$ into their $\frac{dy}{dx}$
$\therefore$ Gradient of normal to curve = $\frac{1}{4}$	B1 ft		Must be numerical $= -1 \div$ their m
$y + 1 = \frac{1}{4}(x - 2)$	M1		Correct equation of straight line through $(2, -1)$, any non-zero numerical gradient
$x - 4y - 6 = 0$	A1	7 7	Correct equation in required form

5.

1 (a)	$y = 2x^3 - 2x^2 - 2x + 8 \Rightarrow \frac{dy}{dx} = 6x^2 - 4x - 2$	M1 A1	1.1b 1.1b
		(2)	
(b)	Attempts $6x^2 - 4x - 2 > 0 \Rightarrow (6x + 2)(x - 1) > 0$	M1	1.1b
	$x = -\frac{1}{3}, 1$	A1	1.1b
	Chooses outside region	M1	1.1b
	$\left\{x : x < -\frac{1}{3}\right\} \cup \{x : x > 1\}$	A1	2.5
		(4)	
			(6 marks)

6.

(i)	Length = 20 – 2x	M1	Expression for length of enclosure in terms of x
	Area = x(20 – 2x) = 20x – 2x²	A1 2	Correctly shows that area = $20x - 2x^2$ **AG**
(ii)	$\frac{dA}{dx} = 20 - 4x$	M1	Differentiates area expression
	For max, 20 – 4x = 0	M1	Uses $\frac{dy}{dx} = 0$
	x = 5 only	A1	
	Area = 50	A1 4	
		6	