Topic X3 Calculus (Post-TT B) [40] MARKSCHEME

1.

$y = 2x + 6x^{-\frac{1}{2}}$	**M1**		Attempt to differentiate
$\frac{dy}{dx} = 2 - 3x^{-\frac{3}{2}}$	**A1**		$kx^{-\frac{3}{2}}$
	A1		Completely correct expression (no +c)
When $x = 4$, gradient $= 2 - \frac{3}{\sqrt{4^3}}$	**M1**		Correct evaluation of either $4^{-\frac{3}{2}}$ or $4^{-\frac{1}{2}}$
$= \frac{13}{8}$	**A1**	**5**	
		[5]	

2.

5 (i)	$\frac{dy}{dx} = -50x^{-6}$	M1		kx^{-6}
		A1	2	Fully correct answer
(ii)	$y = x^{\frac{1}{4}}$	B1		$\sqrt[4]{x} = x^{\frac{1}{4}}$ soi
	$\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$	B1		$\frac{1}{4}x^c$
		B1	3	$kx^{-\frac{3}{4}}$
(iii)	$y = (x^2 + 3x)(1 - 5x)$	M1		Attempt to multiply out fully
	$= 3x - 14x^2 - 5x^3$	A1		Correct expression (may have 4 terms)
	$\frac{dy}{dx} = 3 - 28x - 15x^2$	M1		Two terms correctly differentiated from their expanded expression
		A1	4	Completely correct (3 terms)
			[9]	

3.

8 (i)	$\frac{dy}{dx} = 9 - 6x - 3x^2$	*M1 A1	Attempt to differentiate y or $-y$ (at least one correct term) 3 correct terms
	At stationary points, $9 - 6x - 3x^2 = 0$	M1	Use of $\frac{dy}{dx} = 0$ (for y or $-y$)
	$3(3 + x)(1 - x) = 0$ $x = -3$ or $x = 1$	DM1 A1	Correct method to solve 3 term quadratic $x = -3,\ 1$
	$y = 0,\ 32$	A1ft 6	$y = 0,\ 32$ (1 correct pair www A1 A0)
(ii)	$\frac{d^2y}{dx^2} = -6x - 6$	M1	Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from $k\frac{dy}{dx}$, or other correct method
	When $x = -3, \frac{d^2y}{dx^2} > 0$	A1	$x = -3$ minimum
	When $x = 1, \frac{d^2y}{dx^2} < 0$	A1 3	$x = 1$ maximum
(iii)	$-3 < x < 1$	M1 A1 2	Uses the x values of both turning points in inequality/inequalities Correct inequality or inequalities. Allow $\leq$
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4.

4	$\int_{-2}^{2}(x^4 + 3)\,dx = \left[\frac{1}{5}x^5 + 3x\right]_{-2}^{2}$	M1		Attempt integration – increase of power for at least 1 term
		A1		Obtain correct $\frac{1}{5}x^5 + 3x$
	$= (\frac{32}{5} + 6) - (\frac{-32}{5} - 6)$	M1		Use limits (any two of –2, 0, 2), correct order/subtraction
	$= 24\frac{4}{5}$	A1		Obtain $24\frac{4}{5}$
	area of rectangle = 19 x 4	B1		State or imply correct area of rectangle
	hence shaded area = $76 - 24\frac{4}{5}$	M1		Attempt correct method for shaded area
	$= 51\frac{1}{5}$	A1	7	Obtain $51\frac{1}{5}$ aef such as 51.2, $\frac{256}{5}$
OR				
	Area = $19 - (x^4 + 3)$	M1		Attempt subtraction, either order
	$= 16 - x^4$	A1		Obtain $16 - x^4$ (not from $x^4 + 3 = 19$)
	$\int_{-2}^{2}(16 - x^4)\,dx = \left[16x - \frac{1}{5}x^5\right]_{-2}^{2}$	M1		Attempt integration
		A1		Obtain $\pm\left(16x - \frac{1}{5}x^5\right)$
	$= (32 - \frac{32}{5}) - (-32 - \frac{-32}{5})$	M1		Use limits – correct order / subtraction
	$= 51\frac{1}{5}$	A1		Obtain $\pm 51\frac{1}{5}$
		A1		Obtain $51\frac{1}{5}$ only, no wrong working
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5.

(i)		$\int(2x - 5 + 4x^{-2})\,dx = x^2 - 5x - 4x^{-1}$ $(4a^2 - 10a - {}^2/_a) - (a^2 - 5a - {}^4/_a)$ $= 0$ $3a^2 - 5a + {}^2/_a = 0$ $3a^3 - 5a^2 + 2 = 0$ **AG**	M1	Attempt to rewrite integrand in a suitable form	Attempt to divide all 3 terms by x^2, or attempt to multiply all 3 terms by x^{-2} soi
			A1	Obtain $2x - 5 + 4x^{-2}$	Allow if third term is written in fractional form
			M1	Attempt integration of their integrand	Their integrand must be written as a polynomial ie with all terms of the form kx^n, and no brackets At least two terms must increase in power by 1 Allow if the -5 disappears
			A1	Obtain $x^2 - 5x - 4x^{-1}$	Allow unsimplified (eg ${}^4/_{-1}\,x^{-1}$)
			M1	Attempt use of limits	Must be F(2a) - F(a) ie subtraction with limits in the correct order Allow if no brackets ie $4a^2 - 10a - {}^2/_a - a^2 - 5a - {}^4/_a$ Must be in integration attempt, but allow M1 for limits following M0 for integration eg if fraction not dealt with before integrating
			A1	Equate to 0 and rearrange to obtain $3a^3 - 5a^2 + 2 = 0$	Must be equated to 0 before multiplying through by a At least one extra line of working required between $(4a^2 - 10a - {}^2/_a) - (a^2 - 5a - {}^4/_a) = 0$ and the final answer **AG** so look carefully at working
			[6]		
(ii)		f(1) = 3 – 5 + 2 = 0 **AG** $f(a) = (a - 1)(3a^2 - 2a - 2)$ $a = \frac{2 \pm \sqrt{4+24}}{6} = \frac{2 \pm 2\sqrt{7}}{6} = \frac{1 \pm \sqrt{7}}{3}$ hence $a = {}^1/_3(1 + \sqrt{7})$			**Allow working in x not a throughout**
			B1	Confirm f(1) = 0 – detail required	$3(1)^3 - 5(1)^2 + 2 = 0$ is enough B0 for just f(1) = 0 If using division must show '0' on last line If using coefficient matching must show 'R = 0' If using inspection then there must be some indication of no remainder eg expand to show correct cubic
			M1	Attempt full division by $(a - 1)$, or equiv method	Must be complete method - ie all 3 terms attempted Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of quadratic, considering all relevant terms each time
			A1	Obtain $3a^2$ and one other correct term	Could be middle or final term depending on method Must be correctly obtained Coeff matching - allow for $A = 3$ etc
			A1	Obtain fully correct quotient	Could appear as quotient in long division, or as part of a product if using inspection. For coeff matching it must now be explicit not just $A = 3$, $B = -2$, $C = -2$
			M1	Attempt to solve quadratic	Using the quadratic formula, or completing the square (see guidance sheet) though negative root may be lost at any point M0 if factorising attempt as expected root is a surd Quadratic must come from division attempt, even if this was not good enough for first M1
			A1	Obtain ${}^1/_3(1 + \sqrt{7})$ only	Must give the positive root only, so A0 if negative root still present (but condone $a = 1$ also given) Allow aef but must be a simplified surd as per request on question paper (ie simplify $\sqrt{28}$)
			[6]		