Topic X3 Differentiation (Pre-TT A) [52]

1.

Find $\frac{dy}{dx}$ in each of the following cases.

(i) $y = 5x + 3$ [1]

(ii) $y = \frac{2}{x^2}$ [3]

(iii) $y = (2x + 1)(5x - 7)$ [4]

(Total 8 marks)

2.

Find the gradient of the curve $y = 8\sqrt{x} + x$ at the point whose x-coordinate is 9. [5]

(Total 5 marks)

3.

(i) Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. [6]

(ii) Determine whether each stationary point is a maximum point or a minimum point. [3]

(iii) For what values of x does $x^3 - 3x^2 + 4$ increase as x increases? [2]

(Total 11 marks)

4.

A cuboid has a volume of $8\,m^3$. The base of the cuboid is square with sides of length x metres. The surface area of the cuboid is $A\,m^2$.

(i) Show that $A = 2x^2 + \frac{32}{x}$. [3]

(ii) Find $\frac{dA}{dx}$. [3]

(iii) Find the value of x which gives the smallest surface area of the cuboid, justifying your answer. [4]

(Total 10 marks)

5.

Given that $f(x) = 8x^3 + \frac{1}{x^3}$,

(i) find $f''(x)$, [5]

(ii) solve the equation $f(x) = -9$. [5]

(Total 10 marks)

6.

Diagram not drawn to scale

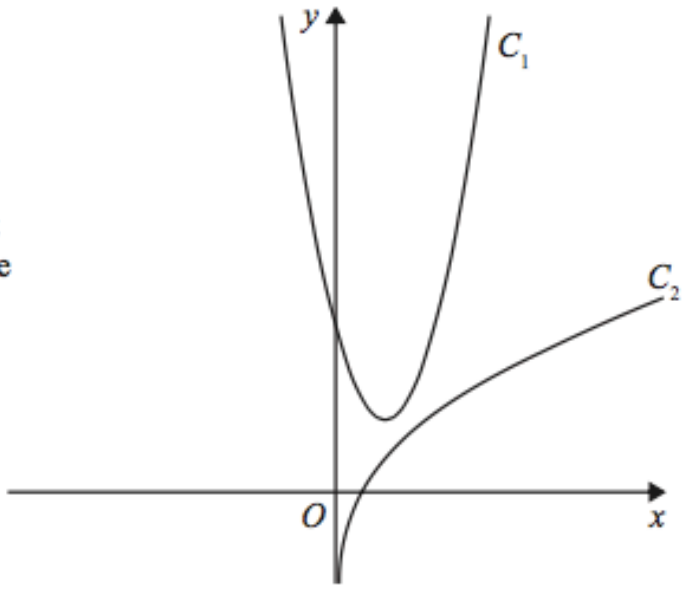

Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

(Total 8 marks)