Topic X3 Differentiation (Pre-TT A) [52] MARKSCHEME

1.

7(i)	$\frac{dy}{dx} = 5$	B1	1	
(ii)	$y = 2x^{-2}$	B1		x^{-2} soi
	$\frac{dy}{dx} = -4x^{-3}$	B1		$-4x^c$
		B1	3	kx^{-3}
(iii)	$y = 10x^2 - 14x + 5x - 7$	M1		Expand the brackets to give an expression of form $ax^2 + bx + c$ $(a \neq 0, b \neq 0, c \neq 0)$
	$y = 10x^2 - 9x - 7$	A1		Completely correct (allow 2 x-terms)
	$\frac{dy}{dx} = 20x - 9$	B1 ft		1 term correctly differentiated
		B1 ft	4	Completely correct (2 terms)
			8	

2.

5		**M1**	Attempt to differentiate
		A1	$kx^{-\frac{1}{2}}$
	$\frac{dy}{dx} = 4x^{-\frac{1}{2}} + 1$	**A1**	
	$= 4\left(\frac{1}{\sqrt{9}}\right) + 1$	**M1**	Correct substitution of $x = 9$ into their
$\frac{dy}{dx}$	$= \frac{7}{3}$	**A1**	$\frac{7}{3}$ only
		5	

3.

(i)	$y = x^3 - 3x^2 + 4$			$3x^2 - 6x$
	$\frac{dy}{dx} = 3x^2 - 6x$	B1		1 term correct
		B1		Completely correct
	$3x^2 - 6x = 0$	M1		$\frac{dy}{dx} = 0$
	$3x(x-2) = 0$	M1		Correct method to solve quadratic
	$x = 0 \quad x = 2$	A1		$x = 0, 2$
	$y = 4 \quad y = 0$	A1√	6	$y = 4, 0$
				SR one correct (x,y) pair **www** **B1**
(ii)	$\frac{d^2y}{dx^2} = 6x - 6$	M1		Correct method to find nature of stationary points (can be a sketch)
	$x = 0 \quad y'' = -6 \quad$ −ve max	B1		$x = 0$ max
	$x = 2 \quad y'' = 6 \quad$ +ve min	B1	3	$x = 2$ min
				(N.B. If no method shown but both min and max correctly stated, award all 3 marks)
(iii)	Increasing	M1		Any inequality (or inequalities) involving both their x values from part (i)
	$x < 0 \quad x > 2$	A1	2	Allow $x \leq 0 \quad x \geq 2$

4.

(i)	Height of box = $\frac{8}{x^2}$	*B1		Area of 1 vertical face $= \frac{8}{x^2} \times x$ $= \frac{8}{x}$
	4 vertical faces $= 4 \times \frac{8}{x}$ $= \frac{32}{x}$	*B1		
	Total surface area = $x^2 + x^2 + \frac{32}{x}$	B1 dep on both **		Correct final expression
	$A = 2x^2 + \frac{32}{x}$		3	
(ii)	$\frac{dA}{dx} = 4x - \frac{32}{x^2}$	B1 B1 B1	3	$4x$ kx^{-2} $-32x^{-2}$
(iii)	$4x - \frac{32}{x^2} = 0$ $4x^3 = 32$ $x = 2$	M1 A1 M1 A1	4	$\frac{dA}{dx} = 0$ soi $x = 2$ Check for minimum Correctly justified **SR** If $x = 2$ stated **www** but with no evidence of differentiated expression(s) having been used in part (iii) **B1**

5.

10(i)	$24x^2 - 3x^{-4}$	B1 B1 B1		$24x^2$ kx^{-4} $-3x^{-4}$
	$48x + 12x^{-5}$	M1 A1	5	Attempt to differentiate their (i) Fully correct
(ii)	$8x^3 + \frac{1}{x^3} = -9$ $8x^6 + 1 = -9x^3$ $8x^6 + 9x^3 + 1 = 0$	*M1		Use a substitution to obtain a 3-term quadratic
	Let $y = x^3$ $8y^2 + 9y + 1 = 0$ $(8y + 1)(y + 1) = 0$	DM1 A1		Correct method to solve quadratic $-\frac{1}{8}$, -1
	$y = -\frac{1}{8}, y = -1$	M1		Attempt to cube root at least one of their y-values
	$x = -\frac{1}{2}, x = -1$	A1	5	$-\frac{1}{2}$, -1
				SR one correct x value **www** **B1**
				SR for trial and improvement: $x = -1$ B1 $x = -\frac{1}{2}$ B2 Justification that there are no further solutions B2
			10	

6.

Question	Scheme	Marks	AOs
15	Finds $\frac{dy}{dx} = 8x - 6$	M1	3.1a
	Gradient of curve at P is -2	M1	1.1b
	Normal gradient is $-\frac{1}{m} = \frac{1}{2}$	M1	1.1b
	So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$	A1	1.1b
	Eliminates y between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in x	M1	3.1a
	Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$	M1	1.1b
	Substitutes to give value for y	M1	1.1b
	Point Q is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$	A1	1.1b
			(8 marks)

Notes:

M1: Differentiates correctly

M1: Substitutes $x = \frac{1}{2}$ to find gradient (may make a slip)

M1: Uses negative reciprocal gradient

A1: Correct equation for normal

M1: Attempts to eliminate y to find an equation in x

M1: Attempts to solve their equation using exp

M1: Uses their x value to find y

A1: Any correct exact form