

### Topic X3 Calculus (Pre-TT B) [42]

1.

The gradient of a curve is given by  $\frac{dy}{dx} = 3x^2 + a$ , where  $a$  is a constant. The curve passes through the points  $(-1, 2)$  and  $(2, 17)$ . Find the equation of the curve. [8]

(Total 8 marks)

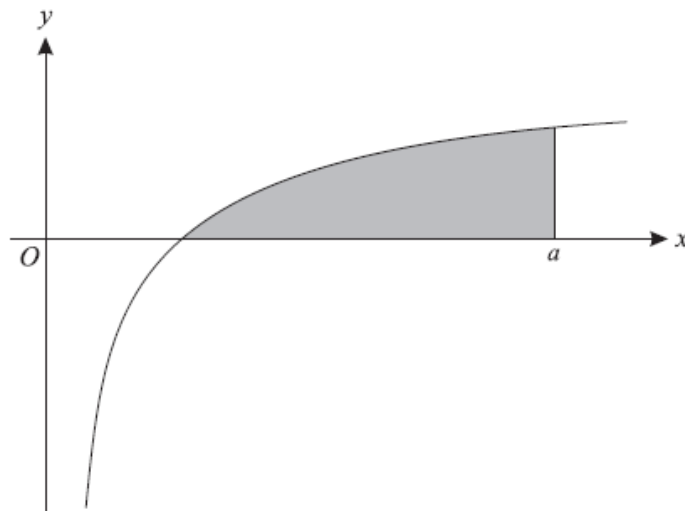
2.

Prove, from first principles, that the derivative of  $3x^2$  is  $6x$ .

(4)

(Total 4 marks)

3.



The diagram shows the graph of  $y = 1 - 3x^{-\frac{1}{2}}$ .

(i) Verify that the curve intersects the  $x$ -axis at  $(9, 0)$ .

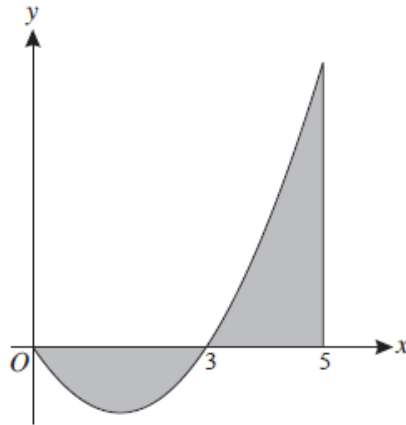
[1]

(ii) The shaded region is enclosed by the curve, the  $x$ -axis and the line  $x = a$  (where  $a > 9$ ). Given that the area of the shaded region is 4 square units, find the value of  $a$ .

[9]

(Total 10 marks)

4.



The diagram shows part of the curve  $y = x^2 - 3x$  and the line  $x = 5$ .

(i) Explain why  $\int_0^5 (x^2 - 3x) dx$  does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

(Total 8 marks)

5.

A curve has equation  $y = x^2 + x$ .

(i) Find the gradient of the curve at the point for which  $x = 2$ . [2]

(ii) Find the equation of the normal to the curve at the point for which  $x = 2$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. [4]

(iii) Find the values of  $k$  for which the line  $y = kx - 4$  is a tangent to the curve. [6]

(Total 12 marks)