Topic X3 Calculus (Pre-TT B) [42] MARKSCHEME

1.

$\int(3x^2 + a)\ \mathrm{d}x = x^3 + ax + c$	M1		Attempt to integrate
	A1		Obtain at least one correct term, allow unsimplified
	A1		Obtain $x^3 + ax$
$(-1, 2) \Rightarrow -1 - a + c = 2$	M1		Substitute at least one of $(-1, 2)$ or $(2, 17)$ into integration attempt involving a and c
$(2, 17) \Rightarrow 8 + 2a + c = 17$	A1		Obtain two correct equations, allow unsimplified
	M1		Attempt to eliminate one variable from two equations in a and c
$a = 2, c = 5$	A1		Obtain $a = 2$, $c = 5$, from correct equations
Hence $y = x^3 + 2x + 5$	A1	8	State $y = x^3 + 2x + 5$

[8]

2.

Question	Scheme	Marks	AOs
6	Considers $\dfrac{3(x+h)^2 - 3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$	M1	1.1b
	So gradient $= \dfrac{6xh + 3h^2}{h} = 6x + 3h$ or $\dfrac{6x\delta x + 3(\delta x)^2}{\delta x} = 6x + 3\delta x$	A1	1.1b
	States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $= 6x$ *	A1*	2.5
		(4 marks)	

Notes:

B1: Gives correct fraction as in the scheme above or $\dfrac{3(x+\delta x)^2 - 3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2 = 3x^2 + 6x\delta x + 3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Uses Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

3.

(i) $0 = 1 - \frac{3}{\sqrt{9}}$	B1	1	Verification of (9, 0), with at least one step shown
(ii) $\int_9^a 1 - 3x^{-\frac{1}{2}}\mathrm{d}x = \left[x - 6\sqrt{x}\right]_9^a$	M1		Attempt integration – increase in power for at least 1 term
	A1		For second term of form $kx^{1/2}$
	A1		For correct integral
$= (a - 6\sqrt{a}) - (9 - 6\sqrt{9})$	M1		Attempt $F(a) - F(9)$
$= a - 6\sqrt{a} + 9$	A1		Obtain $a - 6\sqrt{a} + 9$
$a - 6\sqrt{a} + 9 = 4$	M1		Equate expression for area to 4
$a - 6\sqrt{a} + 5 = 0$	M1		Attempt to solve 'disguised' quadratic
$(\sqrt{a} - 1)(\sqrt{a} - 5) = 0$			
$\sqrt{a} = 1, \sqrt{a} = 5$	A1		Obtain at least $\sqrt{a} = 5$
$a = 1, a = 25$			
but $a > 9$, so $a = 25$	A1	9	Obtain $a = 25$ only
		[10]	

4.

(i)	Some of the area is below the x-axis	B1	1	Refer to area / curve below x-axis or 'negative area'…
(ii)		M1		Attempt integration with any one term correct
		A1		Obtain $^1/_3x^3 - ^3/_2x^2$
	$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3 = \left(9 - \frac{27}{2}\right) - (0 - 0)$	M1		Use limits 3 (and 0) – correct order / subtraction
	$= -4\frac{1}{2}$	A1		Obtain (-)4½
	$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_3^5 = \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right)$	M1		Use limits 5 and 3 – correct order / subtraction
	$= 8\frac{2}{3}$	A1		Obtain $8^2/_3$ (allow 8.7 or better)
	Hence total area is $13^1/_6$	A1	7	Obtain total area as $13^1/_6$, or exact equiv
				SR: if no longer $\int f(x)dx$, then B1 for using [0, 3] and [3, 5]
			8	

5.

10(i)	$\frac{dy}{dx} = 2x + 1$	M1		Attempt to differentiate y
	$= 5$	A1	2	cao
(ii)	Gradient of normal = $-\frac{1}{5}$	B1 ft		ft from a non-zero numerical value in (i)
	When $x = 2, y = 6$	B1		May be embedded in equation of line
	$y - 6 = -\frac{1}{5}(x - 2)$	M1		Equation of line, any non-zero gradient, their y coordinate
	$x + 5y - 32 = 0$	A1	4	Correct equation in correct form
(iii)	$x^2 + x = kx - 4$	*M1		Equating $y_1 = y_2$
	$x^2 + (1 - k)x + 4 = 0$			
	One solution => $b^2 - 4ac = 0$	DM1		Statement that discriminant = 0
	$(1 - k)^2 - 4 \times 1 \times 4 = 0$	DM1		Attempt (involving k) to use a, b, c from their equation
	$(1 - k)^2 = 16$	A1		Correct equation (may be unsimplified)
	$1 - k = \pm 4$	DM1		Correct method to find k, dep on 1st 3Ms
	k = -3 or 5	A1	6	Both values correct
			12	