Topic X3 Calculus (Pre-TT C) [40] MARKSCHEME

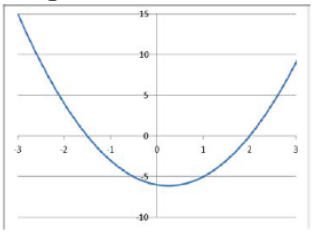
1.

(i)	$\int (2x^2 + 7x + 3)\,dx$ $= \frac{2}{3}x^3 + \frac{7}{2}x^2 + 3x + c$	M1 A1 A1 B1	4	For expanding and integration attempt For at least one term correct For all three terms correct For addition of arbitrary constant, and no $\int$ or dx
(ii)	$\left[2x^{\frac{1}{2}}\right]_0^9$ $= 6$	M1 M1 A1	3 7	For integral of the form $kx^{\frac{1}{2}}$ For evaluating at least F(9) , following attempt at integration For final answer of 6 only

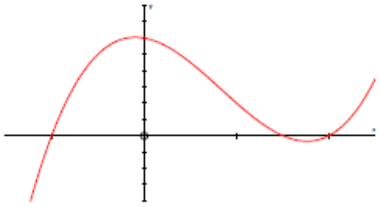
2.

(i)	Intersect where $x^2 + x - 2 = 0 \Rightarrow x = -2, 1$	M1 A1	2	For finding x at both intersections For both values correct
(ii)	Area under curve is $\left[4x - \frac{1}{3}x^3\right]_{-2}^{1}$	M1		For integration attempt with any one term correct
		M1		For use of limits – subtraction and correct order
	i.e. $\left(4 - \frac{1}{3}\right) - \left(-8 + \frac{8}{3}\right) = 9$	A1		For correct area of 9
	Area of triangle is 4½	M1 A1		Attempt area of triangle (½*bh* or integration) Obtain area of triangle as 4 ½
	Hence shaded area is 9 – 4½ = 4½	A1	6	Obtain correct final area of 4 ½
	OR			
	Area under curve is $\int_{-2}^{1}(2 - x - x^2)\,dx$	M1		Attempt subtraction – either order
		M1		For integration attempt with any one term correct
	$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-2}^{1}$	A1		Obtain $\pm\left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]$
	$= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$	M1		For use of limits – subtraction and correct order
	$= 4\frac{1}{2}$	A1		Obtain ± 4 ½ - consistent with their order of subtraction
		A1	8	Obtain 4 ½ only, following correct method only

3.

(i)	$(2x + 3)(x - 2) = 0$ $x = -\frac{3}{2},\ x = 2$ 	M1 A1 B1 B1 B1 [5]	Correct method to find roots Correct roots Reasonably symmetrical positive quadratic curve, must cross x axis y intercept (0, −6) only Good curve, with correct roots indicated and min point in 4th quadrant (not on axis)	Indicated on graph or clearly stated, but there must be a curve Only allow final B1 if curve is clearly intended to be a quadratic symmetrical about min point in 4th quadrant
(ii)	$\frac{dy}{dx} = 4x - 1 = 0$ Vertex when $x = \frac{1}{4}$ $x < \frac{1}{4}$	M1 A1 A1 FT [3]	Attempt to find x coordinate of vertex by differentiating and equating/comparing to zero, completing the square, finding the mid-point of their roots oe cao $x <$ their vertex, allow $\leq$	SC Award **B1 (FT)** for $x < 0$ if clearly from their graph **NB Look for solution to 9ii done in the space below 9i graph**
(iii)	$2x^2 - x - 6 = 4$ $2x^2 - x - 10 = 0$ $(2x - 5)(x + 2) = 0$ $x = \frac{5}{2}, x = -2$ Distance $PQ = 4½$	M1 M1 A1 B1FT [4]	Set quadratic expression equal to 4 Correct method to solve **resulting** three term quadratic Must have both solutions – no mark for one spotted root FT from their x values found from their resulting quadratic, provided $y = 4$	**Not** $2x^2 - x - 6 = 0$ with no use of 4 Allow $\frac{9}{2}$ oe, but do **not** accept unsimplified expressions like $\sqrt{\frac{81}{4}}$

4.

(i)	$-2+k+1+6=0 \Rightarrow k=-5$	M1 A1		For attempting f(–1) For equating f(-1) to 0 and deducing the correct value of k **AG**
	OR	M1 A1		Match coefficients and attempt k Show k = -5
	OR	B2		Following division, state remainder is 0, hence $(x + 1)$ is a factor, hence k = -5
	EITHER: $(x+1)(2x^2-7x+6)$	B1 M1		For correct leading term $2x^2$ For attempt at complete division by f(x) by $(x + 1)$ or equiv.
	$=(x+1)(x-2)(2x-3)$	A1 A1		For completely correct quadratic factor For all three factors correct
	OR: f(2) = 16 – 20 – 2 + 6 = 0 Hence $(x-2)$ is a factor Third factor is $(2x-3)$ Hence f(x) $=(x+1)(x-2)(2x-3)$	M1 A1 M1 A1	6	For further relevant use of the factor theorem For correct identification of factor $(x-2)$ For any method for the remaining factor For all three factors correct
(ii)	$\int_{-1}^{2} f(x)dx = \left[\frac{1}{2}x^4-\frac{5}{3}x^3-\frac{1}{2}x^2+6x\right]_{-1}^{2}$	B1√ B1√		For any two terms integrated correctly For all four terms integrated correctly
	$=\left(8-\frac{40}{3}-2+12\right)-\left(\frac{1}{2}+\frac{5}{3}-\frac{1}{2}-6\right)$	M1		For evaluation of F(2) – F(–1)
	= 9	A1	4	For correct value 9
(iii)		B1 B1	2	For sketch of positive cubic, with three distinct, non-zero, roots For correct explanation that some of the area is below the axis
			12	

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