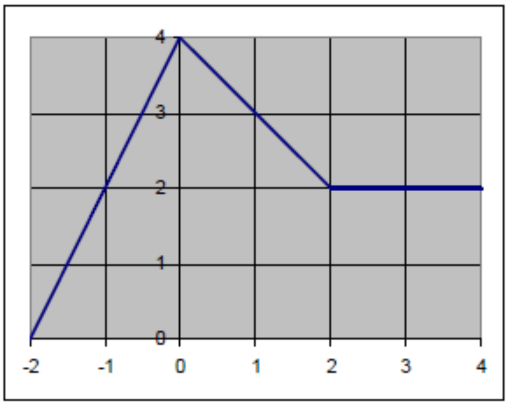
Topic Y1 Polynomials and graphs (Post-TT) [41] MARKSCHEME

1.

Sets $f(-2)=0 \Rightarrow 2\times(-2)^3 - 5\times(-2)^2 + a\times -2 + a = 0$	M1	3.1a
Solves linear equation $2a - a = -36 \Rightarrow a =$	dM1	1.1b
$\Rightarrow a = -36$	A1	1.1b
		(3 marks)

2.

(i)

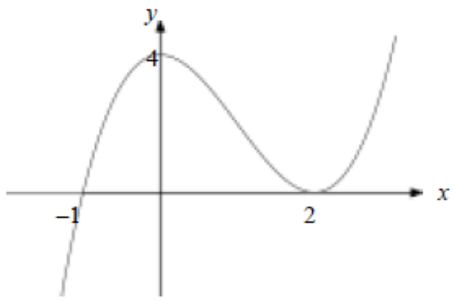


	B1		For $x < 0$, straight line joining (–2, 0) and (0, 4)
	B1	2	For $x > 0$, line joining (0,4) to (2, 2) and horizontal line joining (2,2) and (4,2)
(ii) Translation	**B1**		
1 unit right parallel to x axis	**B1**	2	Allow: 1 unit right, 1 along the x axis, 1 in x **direction**, allow vector notation e.g. $\begin{pmatrix}1\\0\end{pmatrix}$, 1 unit horizontally
		4	

3.

(i) $(x^2-4x+4)(x+1)$	**M1**		Attempt to multiply a 3 term quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term)
	A1		Expansion with at most 1 incorrect term
$= x^3 - 3x^2 + 4$	**A1**	3	Correct, simplified answer

(ii)

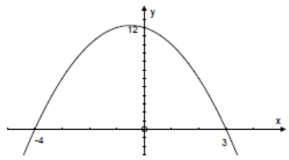


B1		+ve cubic with 2 or 3 roots
B1		Intercept of curve labelled (0, 4) or indicated on y-axis
B1	3	(–1, 0) and turning point at (2, 0) labelled or indicated on x-axis and no other x intercepts
	6	

4.

<u>Method 1 (Long division)</u>		
Clear correct division method at beginning	M1	x^2 in quot, mult back & attempt subtraction [At subtraction stage, cf $(x^4) = 0$]
Correct method up to & including x term in quot	M1	[At subtraction stage, cf $(x^3) = 0$]
<u>Method 2 (Identity)</u>		
Writing $(x^2 + 2x - 1)(x^2 + bx + 2) + cx + 7$	M1	Probably equated to $x^4 - 2x^3 - 7x^2 + 7x + a$
Attempt to compare cfs of x^3 or x^2 or x or const	M1	
Then:		
$b = -4$	A1	
$c = -1$	A1	
$a = 5$	A1 **5**	

5.

(i)		$(x-3)(x+4) = 0$ $x = 3$ or $x = -4$ 	M1 A1 B1 B1 B1 [5]	Correct method to find roots Correct roots Negative quadratic curve y intercept (0, 12) Good curve, with correct roots 3 and –4 indicated and max point in 2[nd] quadrant	i.e. max at (0, 12) **B0** Curve must go below x-axis for final mark
(ii)		$-4 < x < 3$	M1 A1 [2]	Correct method to solve quadratic inequality Allow $\le$ for the method mark but not the accuracy mark	their lower root $< x <$ their higher root Allow "$x > -4, x < 3$" Allow "$x > -4$ and $x < 3$" Do not allow "$x > -4$ **or** $x < 3$"
(iii)		$y = 4 - 3x$ $12 - x - x^2 = 4 - 3x$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4$ or $x = -2$ $y = -8$ or $y = 10$	M1 A1 M1 A1 A1 [5]	substitute for x/y or attempt to get an equation in 1 variable only obtain correct 3 term quadratic correct method to solve 3 term quadratic	e.g. for first mark $3x + 12 - x - x^2 = 4$, or $y = 12 - \left(\frac{4-y}{3}\right) - \left(\frac{4-y}{3}\right)^2$ (this leads to $y^2 - 2y - 80 = 0$). Condone poor algebra for this mark. **SC** If **A0 A0**, give **B1** for one correct pair of values spotted or from correct factorisation www

6.

(i)		$f(2) = 8 + 2a - 6 + 2b = 0$ $g(2) = 24 + 4 + 10a + 4b = 0$	M1	Attempt at least one of f(2), g(2)	Allow for substituting $x = 2$ into either equation – no need to simplify at this stage. Division – complete attempt to divide by $(x - 2)$. Coeff matching - attempt all 3 coeffs of quadratic factor.
			M1	Equate at least one of f(2) and g(2) to 0	Just need to equate their substitution attempt to 0 (but just writing eg f(2) = 0 is not enough). It could be implied by later working, even after attempt to solve equations. Division - equating their remainder to 0. Coeff matching – equate constant terms.
		$2a + 2b = -2$, $5a + 2b = -14$	A1	Obtain two correct equations in a and b	Could be unsimplified equations. Could be $8a + 2b = -26$ (from f(2) = g(2)).
		hence $3a = -12$	M1	Attempt to find a (or b) from two simultaneous eqns	Equations must come from attempts at two of f(2) = 0, g(2) = 0, f(2) = g(2). M1 is awarded for eliminating a or b from 2 sim eqns – allow sign slips only. Most will attempt a first, but they can also gain M1 for finding b from their simultaneous equations.
		so $a = -4$ **AG**	A1	Obtain $a = -4$, with necessary working shown	If finding b first, then must show at least one line of working to find a (unless earlier shown explicitly eg $a = -1 - b$).
		$b = 3$	A1	Obtain $b = 3$	Correct working only
			[6]		**SR Assuming a = -4** **Either use this scheme, or the original, but don't mix elements from both** M1 Attempt either f(2) or g(2) M1 Equate f(2) or g(2) to 0 (also allow f(2) = g(2)) A1 Obtain $b = 3$ A1 Use second equation to confirm $a = -4$, $b = 3$
(ii)		$f(x) = (x - 2)(x^2 + 2x - 3)$ $= (x - 2)(x + 3)(x - 1)$	M1	Attempt full division of their f(x) by $(x - 2)$ Could also be for full division attempt by $(x - 1)$ or $(x + 3)$ if identified as factors	Must be using $f(x) = x^3 - 7x + k$. Must be complete method – ie all 3 terms attempted. Long division – must subtract lower line (allow one slip). Inspection – expansion must give at least three correct terms of their cubic. Coefficient matching – must be valid attempt at all 3 quadratic coeffs, considering all relevant terms each time. Factor theorem – must be finding 2 more factors / roots.
			A1	Obtain x^2 and at least one other correct term, from correct f(x)	Could be middle or final term depending on method. Coeff matching – allow for stating values eg $A = 1$ etc. Factor theorem – state factors of $(x + 3)$ and $(x - 1)$.
			A1	Obtain $(x - 2)(x + 3)(x - 1)$	Must be seen as a product of three linear factors. Answer only gains all 3 marks.
		$g(x) = (x - 2)(3x^2 + 7x - 6)$ $= (x - 2)(x + 3)(3x - 2)$ **OR** $g(1) = -4$, $g(-3) = 0$	M1	Attempt to verify two common factors	Possible methods are: Factorise g(x) completely – f(x) must have been factorised. Find quadratic factor of g(x) and identify $x = -3$ as root. Test their roots of f(x) in g(x). Just stating eg g(-3) = 0 is not enough – working required. If f(x) hasn't been factorised, allow M1 for using factor thm on both functions to find common factor, or for factorising g(x) and testing roots in f(x).
		Hence common factor of $(x + 3)$	A1	Identify $(x + 3)$ as a common factor	Just need to identify $(x + 3)$ - no need to see $(x - 2)$ or to explicitly state 'two common factors'. Need to see $(x + 3)$ as factor of g(x) – just showing g(-3) = 0 and then stating 'common factor' is not enough. CWO (inc A0 for $g(x) = (x - 2)(x + 3)(x - {}^2/_3)$). If using factor thm, no need to find g(1) if g(-3) done first. Just stating $(x + 3)$ with no supporting evidence is M0A0. A0 if referring to -3 (and 2) as 'factors'. A0 if additional incorrect factor given.
			[5]		